



كلية العلوم

القسم : الرياضيات

السنة : الثانية

المادة : معادلات تفاضلية 2

المحاضرة : الثالثة/ عملي/

{{ مكتبة A to Z }}

مكتبة A to Z Facebook Group :

كلية العلوم

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر (What's app-Telegram) على الرقم 0931497960

2026



الدكتور:

المحاضرة:

3 عماديه



القسم: الرياضيات

السنة: الثانية

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A to Z Library for university services

$$(1) \quad x^2 y'' + x(3+x)y' + (1+x)y = 0 \quad \text{في } x=0$$

نقسم على x^2 فنحصل على

$$y'' + \left(\frac{3}{x} + 1\right)y' + \left(\frac{1}{x} + 1\right)y = 0$$

$x=0$ نقطة حادة للمعادلة لأننا نأخذ Q و P

$x=0$ نقطة نظامية:

$$y = \sum_{n=0}^{\infty} C_n x^{n+x} \quad y' = \sum_{n=0}^{\infty} (n+x) C_n x^{n+x-1} \quad y'' = \sum_{n=0}^{\infty} (n+x)(n+x-1) C_n x^{n+x-2}$$

ننوّض في المعادلة:

$$x^2 \sum_{n=0}^{\infty} (n+x)(n+x-1) C_n x^{n+x-2} + (3x+x^2) \sum_{n=0}^{\infty} (n+x) C_n x^{n+x-1} + (1+x) \sum_{n=0}^{\infty} C_n x^{n+x} = 0$$

$$= \sum_{n=0}^{\infty} (n+x)(n+x-1) C_n x^{n+x} + \sum_{n=0}^{\infty} 3(n+x) C_n x^{n+x} + \sum_{n=0}^{\infty} (n+x) C_n x^{n+x+1}$$

$$+ \sum_{n=0}^{\infty} C_n x^{n+x} + \sum_{n=0}^{\infty} C_n x^{n+x+1} = 0$$

$$= \sum_{n=0}^{\infty} [(n+x)(n+x-1) + 3(n+x) + 1] C_n x^{n+x} + \sum_{n=1}^{\infty} [(n-1+x)+1] C_n x^{n+x}$$

نفرض $C_0 = 0$

$$(x+1)^2 = 0 \Rightarrow x = -1$$





$$\sum_{n=0}^{\infty} C_n (n+k+1)^2 x^{n+k} + \sum_{n=0}^{\infty} C_{n-1} (n+k) x^{n+k} = 0$$

$$C_n (n+k+1)^2 + C_{n-1} (n+k) = 0$$

$$C_n = - \frac{n+k}{(n+k+1)^2} C_{n-1}; \quad n \geq 1$$

← n=1

$$C_1 = - \frac{1+k}{(2+k)^2} C_0$$

$$C_2 = - \frac{2+k}{(k+3)^2} C_1 \quad \leftarrow n=2$$

$$C_3 = - \frac{3+k}{(k+4)^2} C_2 \quad \leftarrow n=3$$

$$\Rightarrow C_3 = - \frac{3+k}{(k+4)^2} C_2 = - \frac{1+k}{(k+2)(k+3)(k+4)^2}$$

$$\bar{y} = x^k \sum_{n=0}^{\infty} C_n x^n = x^k \left(1 + \frac{1+k}{(k+2)^2} x + \frac{1+k}{(k+2)(k+3)^2} x^2 + \dots \right)$$

$$y_1 = \bar{y} \Big|_{x=1} = x^{-1} \cdot x^1 = \frac{1}{x}$$

$$y_2 = y_1 \int \frac{P^{-k} dx}{y^2} \cdot P^{-k} dx = \int \frac{P^{-k+1} dx}{x^2} = \int \frac{1}{x^2} dx = -\frac{1}{x} = -x^{-1}$$

$$y_2 = \frac{1}{x} \int \frac{P^{-k}}{1/x^2} dx = \frac{1}{x} \int \frac{P^{-k}}{x} dx$$

$$y_2 = \frac{1}{x} \int \frac{1}{x} \left[1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right] dx$$



$$= \frac{1}{x} \int \left[\frac{1}{x} - 1 + \frac{x}{2!} - \frac{x^2}{3!} + \dots \right] dx = \frac{1}{x} \left[\ln x - x + \frac{x^2}{4} + \frac{x^3}{18} + \dots \right]$$

$y = Ay_1 + By_2$

②. $xy'' - (4+x)y' + 2y = 0$; $x=0$

في y'' الدالة هي

$$y'' - \left(\frac{4}{x} + 1\right)y' + \frac{2}{x}y = 0$$

بما أن q, p دالة في $x=0$ فإننا نستخدم طريقة كوتاني $x=0$

$$y = \sum_{n=0}^{\infty} C_n x^{n+k}, \quad y' = \sum_{n=0}^{\infty} (n+k) C_n x^{n+k-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+k)(n+k-1) C_n x^{n+k-2}$$

$$\sum_{n=0}^{\infty} (n+k-1)(n+k) C_n x^{n+k-1} - 4 \sum_{n=0}^{\infty} (n+k) C_n x^{n+k-1} - \sum_{n=0}^{\infty} (n+k) C_n x^{n+k} + 2 \sum_{n=0}^{\infty} C_n x^{n+k} = 0$$

$$\sum_{n=0}^{\infty} ((n+k-1-4)(n+k)) C_n x^{n+k-1} - \sum_{n=0}^{\infty} (n+k-2) C_n x^{n+k}$$

$$(k-5)(k) C_0 x^{k-1} + \sum_{n=1}^{\infty} (n+k-5)(n+k) C_n x^{n+k-1} - \sum_{n=1}^{\infty} (n+k-3) C_n x^{n+k} = 0$$

← $C_0 \neq 0$

$k(k-5) = 0 \Rightarrow k_1 = 0$ أو $k_2 = 5$



$$\sum_{n=1}^{\infty} (n+x-5)(n+x) C_n x^{n+x} - \sum_{n=1}^{\infty} (n+x-3) C_{n-1} x^{n+x-1} = 0$$

$$(n+x-5)(n+x) C_n - (n+x-3) C_{n-1} = 0$$

$$C_n = \frac{(n+x-3)}{(n+x-5)(n+x)} C_{n-1}$$

$x=0$ نفي

$$C_n = \frac{n-3}{n(n-5)} C_{n-1}$$

نفي C_0 $n=5$ اطلاق

$$C_1 = \frac{x-2}{(x-4)(x+1)} C_0$$

$$C_2 = \frac{x-1}{(x-3)(x+2)} C_1 = \frac{(x-1)(x-2)}{(x-3)(x+2)(x-4)(x+1)}$$

$$\bar{y} = x^x \left(C_0 + \frac{x-2}{(x-4)(x+1)} C_0 x + \frac{(x-1)(x-2)}{(x-3)(x+2)(x-4)(x+1)} C_0 x^2 + \dots \right)$$

$$\frac{\partial \bar{y}}{\partial x} = x^x \left[\ln x \left(C_0 + \frac{x-2}{(x-4)(x+1)} C_0 x + \frac{(x-1)(x-2)}{(x-3)(x+2)(x-4)(x+1)} C_0 x^2 \right) \right]$$

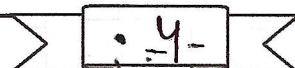
$$+ x^x \left(\frac{-x^2+4x-10}{(x-4)^2(x+1)^2} x + \dots \right)$$

$C_0 = x$ ← $C_0 = x - x_0$ نفي

$$\bar{y} = x^x \left(x + \frac{x-2}{(x-4)(x+1)} x x + \dots \right)$$

$$\frac{\partial \bar{y}}{\partial x} = x^x \left[\ln \left[x + \frac{x(x-2)}{(x-4)(x+1)} x + \dots \right] + x^x \left[\frac{-x^2+4x-10}{(x-4)^2(x+1)^2} x + \dots \right] \right]$$

$$y_1 = \bar{y} \Big|_{x=0} \quad y_2 = \frac{\partial \bar{y}}{\partial x} \Big|_{x=0} \Rightarrow y = A y_1 + B y_2$$





③. $x y'' - (x+1) y' - y = 0$

$$y = \sum_{n=0}^{\infty} C_n \cdot x^{n+k}, \quad y' = \sum_{n=0}^{\infty} (n+k) C_n \cdot x^{n+k-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+k-1)(n+k) C_n \cdot x^{n+k-2}$$

$$\sum_{n=0}^{\infty} (n+k)(n+k-1) C_n \cdot x^{n+k-1} - \sum_{n=0}^{\infty} (n+k) C_n \cdot x^{n+k} - \sum_{n=0}^{\infty} (n+k) C_n \cdot x^{n+k-1}$$

$$- \sum_{n=0}^{\infty} C_n \cdot x^{n+k} = 0$$

$$= \sum_{n=0}^{\infty} (n+k)(n+k-2) C_n \cdot x^{n+k-1} - \sum_{n=0}^{\infty} (n+k) C_{n-1} \cdot x^{n+k-1} = 0$$

$$n=0 \Rightarrow x(x-2) = 0 \Rightarrow x_1 = 0, x_2 = 2$$

$$C_n = \frac{1}{n+k-2} C_{n-1}$$

$$x=0 \Rightarrow C_n = \frac{1}{n-2} C_{n-1}$$

← $C_0 = x$ → غير صحيح C_2

$$C_1 = \frac{1}{x-1} C_0, \quad C_0 = \frac{x}{x-1}$$

$$C_2 = \frac{1}{x} C_1 = \frac{1}{x(x-1)} (x) = \frac{1}{x-1}$$

$$C_3 = \frac{1}{x+1} C_2 = \frac{1}{(x+1)(x-1)}$$

$$\bar{y} = x^x \left[x + \frac{x}{x-1} x + \frac{1}{x-1} x^2 + \frac{1}{x^2-1} x^3 + \dots \right]$$



$$y_1 = \bar{y} \Big|_{x=0}$$

$$y_1 = 1 [-x^2 - x^3 \dots]$$

$$\frac{\partial \bar{y}}{\partial x} = x^x \ln \left[x + \frac{x}{x-1} x + \dots \right] + x^x \left[1 - \frac{1}{(x-1)^2} x - \frac{1}{(x-1)^2} x^2 + \frac{2x}{(x-1)^2} x^3 + \dots \right]$$

$$y_2 = \frac{\partial \bar{y}}{\partial x} \Big|_{x=0} = \ln x [-x^2 - x^3 \dots] + \left[1 - x - x^2 + \frac{1}{4} x^4 \dots \right]$$

$$y = Ay_1 + By_2$$

(5)

$$4x^3 y'' + 6x^2 y' + y = 0$$

$$: y'x = -y'_t t^2 \quad \leftarrow x = \frac{1}{t} \text{ نضع}$$

$$y''x = t^4 \cdot y''_t + 2t^3 y'_t$$

$$4(t y'' + 2y'_t) - 6y'_t + y = 0$$

$$4t y'' + 2y'_t + y = 0$$

تكون نقطة حالة نظام

محلولة بالنظرية:

$$y = A \left[1 - \frac{1}{2} t + \dots \right] + B t^{\frac{1}{2}} \left[1 - \frac{1}{6} t + \dots \right] \text{ بالنظرية}$$

$$A = \left[1 - \frac{1}{2} x^{-1} + \dots \right] + B x^{-\frac{1}{2}} \left[1 - \frac{1}{6} x^{-1} + \dots \right]$$

انتبه الحاصلة



مكتبة AZ to Z