كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : معادلات تفاضلية2

المحاضرة : الثانية/عملي/

{{ A to Z مكتبة }}

Facebook Group : A to Zمكتبة

① $xy'' - y' - 4x^3 y = 0$

نقسم على أمثال y''

$y'' - \frac{1}{x} y' - 4x^2 y = 0$

$P(x) = -\frac{1}{x}$, $q = -4x^2 \Rightarrow q' = -8x$

$\frac{q' + 2Pq}{q\sqrt{q}} = \frac{-8x + 8x}{q\sqrt{q}} = 0$ (ثابت)

$k = -1 \Rightarrow Z'_x = \sqrt{kq} = \sqrt{4x^2} = 2x$

$Z_x = x^2 \Rightarrow y'_x = \frac{dy}{dx} = \frac{dy}{dZ} \cdot \frac{dZ}{dx}$

$y'_x = y'_Z \cdot Z'_x = y'_Z \cdot 2x$

$y''_x = (Z'_x)^2 y''_Z + y'_Z \cdot Z''_x = 4x^2 y''_Z + 2y'_Z$

$4x^2 y''_Z + 2y'_Z - 2y'_Z - 4x^2 y = 0$

$y''_Z - y = 0$

$m^2 - 1 = 0 \Rightarrow m = \mp 1$

$y_h = C_1 e^{-Z} + C_2 e^{Z}$

$y = C_1 e^{-x^2} + C_2 e^{x^2}$

$(1-x^2)y'' - xy' - 9y = 0$

نقسم على امثال y''

$$y'' - \frac{x}{1-x^2}y' - \frac{9}{1-x^2}y = 0$$

$$P = -\frac{x}{1-x^2} , \quad q = -\frac{9}{1-x^2}$$

$$\frac{q' + 2Pq}{q\sqrt{q}} = \frac{\frac{-18}{(1-x^2)^2} + \frac{18}{(1-x^2)^2}}{q\sqrt{q}} = 0$$

$$Z'_x = \sqrt{q}\,k \;;\; k = -1$$

$$Z'_x = \sqrt{\frac{9}{1-x^2}} = \frac{3}{\sqrt{1-x^2}}$$

$$Z_x = 3\arcsin x$$

$$y'_x = \frac{dy}{dx} = \frac{dy}{dZ}\frac{dZ}{dx} = y'_Z \frac{3}{\sqrt{1-x^2}}$$

$$y''_x = y'_Z \cdot Z''_x + Z'^2_x \cdot y''_Z$$

$$y''_x = \frac{3x}{\sqrt{1-x^2}\,(1-x^2)} \cdot y'_Z + \frac{9}{1-x^2} \cdot y''_Z$$

نعوض بالمعادلة:

$$\cancel{\frac{3x}{\sqrt{1-x^2}}\,y'_Z} + 9y''_Z - \cancel{\frac{3x}{\sqrt{1-x^2}}\,y'_Z} - 9y = 0$$

$$y'' - y = 0$$

$$m^2 - 1 = 0 \Rightarrow m_1 = 1 , m_2 = -1 \Rightarrow y_h = C_1 e^{Z} + C_2 e^{-Z} = C_1 e^{3\arcsin x} + C_2 e^{-3\arcsin x}$$

3) $y'' - \tan x \, y' + \sec^2 y = 0$

$P(x) = -\tan x, \quad q = \sec^2 = \frac{1}{\cos^2 x}$

$$q'(x) = \frac{2\sin x \cos x}{\cos^4 x} = \frac{2\sin x}{\cos^3 x}$$

$$\frac{q' + 2Pq}{q\sqrt{q}} = \frac{\frac{2\sin x}{\cos^3 x} - \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos^2 x}}{q\sqrt{q}} = 0$$

$$Z'_x = \sqrt{qk} = \sqrt{\frac{1}{\cos^2 x}} \; ; \; q = 1$$

$$Z'_x = \frac{1}{\cos x} \Longrightarrow Z_x = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$$

$$y''_x = y''_Z \cdot Z'^2_x + y'_Z \cdot Z''_x$$

$$y'_x = Z'_x \cdot y'_Z$$

$$y''_x = y''_Z \cdot \frac{1}{\cos^2 x} + y'_Z \cdot \frac{\sin x}{\cos x}, \quad y'_x = \frac{1}{\cos x} y'_Z$$

نعوض بالمعادلة:

$$\frac{1}{\cos^2 x} y''_Z + \frac{\sin x}{\cos^2 x} y'_Z - \frac{\sin x}{\cos^2 x} y'_Z + \frac{1}{\cos^2 x} y = 0$$

$$y'' + y = 0$$

$$m^2 + 1 = 0 \Longrightarrow m = \mp i$$

$$y_h(z) = C_1 \cos Z_1 + C_2 \sin(Z_1)$$

$$y(x) = C_1 \cos x \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right) + C_2 \sin\left(\ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)\right)$$

(4) $(1-x^2)y'' + xy' - y = 0 \,;\, x=0$

$x=0$ نقطة عادية للمعادلة لأنها عادية لـ p و q

$y = \sum_{n=0}^{\infty} C_n x^n \,,\; y' = \sum_{n=0}^{\infty} n C_n x^{n-1} \,,\; y'' = \sum_{n=0}^{\infty} n(n-1) C_n x^{n-2}$

نعوض بالمعادلة:

$$(1-x^2)\sum_{n=0}^{\infty} n(n-1) C_n x^{n-2} + x\sum_{n=0}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=0}^{\infty} n(n-1) C_n x^{n} + \sum_{n=0}^{\infty} n C_n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) C_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1) C_n x^n + \sum_{n=0}^{\infty} n C_n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$= (n+2)(n+1) C_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1) C_n x^n + \sum_{n=0}^{\infty} n C_n x^n - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$(n+2)(n+1) C_{n+2} - \left(n(n-1) - (n-1)\right) C_n = 0$$

$$C_{n+2} = \frac{(n-1)^2}{(n+2)(n+1)} C_n$$

$n=0 \Longrightarrow C_2 = \frac{1}{2} C_0$

$n=1 \Longrightarrow C_3 = 0$

$n=2 \Longrightarrow C_4 = \frac{1}{12} C_2 = \frac{1}{24} C_0$

$n=3 \Longrightarrow C_5 = 0$

$$y = C_0 + C_1 x + \frac{1}{2} C_0 x^2 + \frac{1}{24} C_0 x^4 + \cdots$$

$$y = C_1 x + C_0\left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots\right) = A y_1 + B y_2$$

5. $y'' + (x-1)y' + y = 0 \; ; \; x = 2$

$x = 2$ نقطة عادية

$P_x = x - 1 \; , \; q = 1$

نفرض $t = x - 2 \Leftarrow t = 0$

$y''_x = y''_t \; , \; y'_x = y'_t$

$y'' + (t+1)y' + y = 0$

$y = \sum_{n=0}^{\infty} C_n t^n$

$y' = \sum_{n=0}^{\infty} n C_n t^{n-1} \; , \; y'' = \sum_{n=0}^{\infty} n(n-1) C_n t^{n-2}$

نعوض بالمعادلة

$\sum_{n=0}^{\infty} n(n-1) t^{n-2} + (t+1) \sum_{n=0}^{\infty} n C_n t^{n-1} + \sum_{n=0}^{\infty} C_n t^n$

$\sum_{n=0}^{\infty} n(n-1) C_n t^{n-2} + \sum_{n=0}^{\infty} n C_n t^n + \sum_{n=0}^{\infty} n C_n t^{n-1} + \sum_{n=0}^{\infty} C_n t^n$

$\sum_{n=0}^{\infty} (n+2)(n+1) C_{n+2} t^n + \sum_{n=0}^{\infty} n C_n t^n + \sum_{n=0}^{\infty} (n+1) C_{n+1} t^n + \sum_{n=0}^{\infty} C_n t^n$

$(n+2)(n+1) C_{n+2} + n C_n + (n+1) C_{n+1} + C_n = 0$

$C_{n+2} = \frac{(n+1) C_{n+1} + (n+1) C_n}{(n+2)(n+1)} = \frac{C_{n+1} + C_n}{-n+2}$

$n = 0 \Rightarrow C_2 = -\frac{1}{2} C_1 - \frac{1}{2} C_0$

$n = 1 \Rightarrow C_3 = -\frac{1}{3} C_2 - \frac{1}{3} C_1 = +\frac{1}{6} C_1 + \frac{1}{6} C_0 - \frac{1}{3} C_1$

$= \frac{1}{6} C_0 - \frac{1}{6} C_1$

$n=2 \Rightarrow C_4 = -\frac{1}{4}C_3 - \frac{1}{4}C_2$

$C_4 = -\frac{1}{24}C_0 + \frac{1}{24}C_1 + \frac{1}{8}C_1 + \frac{1}{8}C_0 = \frac{1}{12}C_0 + \frac{1}{6}C_1$

$y = \sum_{n=0}^{\infty} C_n t^n$

$y = C_0 + C_1 t - \frac{1}{2}C_1 t^2 - \frac{1}{2}C_0 t^2 + \frac{1}{6}C_0 t^3 - \frac{1}{6}C_1 t^3 + \frac{1}{2}C_0 t^4 + \frac{1}{6}C_1 t^4$

$y = C_0\left(1 - \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{12}t^4 + \cdots\right) + C_1\left(t - \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{6}t^4 + \cdots\right)$

$y = C_0\left(1 - \frac{1}{2}(x-2)^2 + \frac{1}{6}(x-2)^3 + \frac{1}{12}(x-2)^4 + \cdots\right) + C_1\left((x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{6}(x-2)^3 + \frac{1}{12}(x-2)^4 + \cdots\right)$

⑥ $(x-1)y'' - xy' + y = 0 \; ; \; x=0$

$x=0$ نقطة عادية للمعادلة لأنها عادية لـ P و q

نفرض:

$y = \sum_{n=0}^{\infty} C_n x^n , \quad y' = \sum_{n=0}^{\infty} n C_n x^{n-1} , \quad y'' = \sum_{n=0}^{\infty} n(n-1) C_n x^{n-2}$

نعوض بالمعادلة:

$(x-1)\sum_{n=0}^{\infty} n(n-1)C_n x^{n-2} - x\sum_{n=0}^{\infty} n C_n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n$

$\sum_{n=0}^{\infty} n(n-1)C_n x^{n-1} - \sum_{n=0}^{\infty} n(n-1)C_n x^{n-2} - \sum_{n=0}^{\infty} n C_n x^n$

$+ \sum_{n=0}^{\infty} C_n x^n = 0$

$$\sum_{n=0}^{\infty}(n+1)(n)C_{n+1}x^n - \sum_{n=0}^{\infty}(n+2)(n+1)C_{n+2}x^n - \sum_{n=0}^{\infty}C_n(n-1)x^n = 0$$

$$n(n+1)C_{n+1} - (n+2)(n+1)C_{n+2} - (n-1)C_n = 0$$

$$C_{n+2} = \frac{n}{n+2}C_{n+1} - \frac{n-1}{(n+2)(n+1)}C_n$$

$n=0 \implies C_2 = -\frac{1}{2}C_0$

$n=1 \implies C_3 = \frac{1}{3}C_2 = -\frac{1}{6}C_0$

$n=2 \implies C_4 = \frac{2}{4}C_3 - \frac{1}{12}C_2 = -\frac{1}{12}C_0 + \frac{1}{24}C_0 = -\frac{1}{24}C_0$

$$y = \sum_{n=0}^{\infty}C_n x^n$$

$$= C_0 + C_1x - \frac{1}{2}C_0x^2 - \frac{1}{6}C_0x^3 - \frac{1}{24}C_0x^4 + \cdots$$

$$y = C_1x - C_0\left(1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{24}x^4 + \cdots\right)$$

6

صنف النقاط الشاذة للمعادلة

$$x(x-1)^2(x+2)y'' + x^2y' - (x^3+2x-1)y = 0$$

نقسم على امثال y'':

$$y'' + \frac{x}{x(x-1)^2(x+2)}y' - \frac{x^3+2x-1}{x(x-1)^2(x+2)}$$

النقاط الشاذة هي $x=0$ و $x=1$ و $x=-2$

$x=0$ عادية: $x=0 \implies (x-x_0)P_{(x)} = \frac{x^2}{(x-1)^2(x+2)}$

$x=0 \Rightarrow (x-x_0)^2 q(x)$

$= \dfrac{x(x^3+2x-1)}{(x-1)^2(x+2)}$

$x=0$ نقطة شاذة نظامية

$x=1$

$(x-1)P(x) = \dfrac{x}{(x-1)(x+2)}$

ليست نقطة شاذة نظامية

$x=-2$

$(x+2)P(x) = \dfrac{x}{(x-1)^2}$ عادية

$(x+2)^2 q(x) = \dfrac{-x^3+2x-1}{x(x-1)^2}(x+2)$ = عادية

$x=-2$ حالة

انتهت المحاضرة