كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : معادلات تفاضلية2

المحاضرة : الاولى/**عملي**/

{{ A to Z مكتبة }}

① $x^2y'' + 5xy' + 3y = 8x + 15x^2\ln x$

نفرض $x = e^t \Leftarrow t = \ln(x)$

$xy' = y'_t$, $x^2y'' = y''_t - y'_t$

$y''_t - y'_t + 5y'_t + 3y = 8e^t + 15te^{2t} \Rightarrow$

$y''_t + 4y'_t + 3y = 8e^t + 15te^{2t} \Rightarrow$ نكتب المعادلة المميزة:

$m^2 + 4m + 3 = 0$

$(m+1)(m+3) = 0 \Rightarrow m_1 = -1, m_2 = -3$

$y_h = c_1 e^{-x} + c_2 e^{-3x}$

$y_p = Ae^t + Bte^{2t} + Ce^{2t}$ .... $\times 3$

$y'_p = Ae^t + Be^{2t} + 2Bte^{2t} + 2Ce^{2t}$ .... $\times 4$

$y''_p = Ae^t + Be^{2t} + 2Be^{2t} + 4Bte^{2t} + 4Ce^{2t}$ .... $\times 1$

بالضرب ثم الجمع للحدود المتشابهة نجد أن:

$8Ae^t + 8Be^{2t} + 15Bte^{2t} + 15Ce^{2t} = 8e^t + 15e^{2t}$

$8A = 8 \Rightarrow A = 1$; $15B = 15 \Rightarrow B = 1$

$8B + 15C = 0 \Rightarrow C = \frac{-8}{15}$

نعوض في y_p:

$y_p = e^t + te^{2t} - \frac{8}{15}e^{2t} = x + x^2\ln x - \frac{8}{15}x^2$

$y = y_p + y_h = \frac{1}{x}c_1 + \frac{c_2}{x^3} + x + x^2\ln x - \frac{8}{15}x^2$

② $x^2y'' + xy' - 4y = \ln x$

نفرض $x = e^t \Leftarrow t = \ln x$

$xy' = y'_t$ ، $x^2y'' = y''_t - y'_t$

نعوّض بالمعادلة:

$y''_t - y'_t + y'_t - 4y = \ln x$

$y''_t - 4y = \ln e^t$

$m^2 - 4 = \ln e^t$

$(m^2 - 4) = 0 \Rightarrow (m-2)(m+2) = 0 \Rightarrow m_1 = 2 , m_2 = -2$

$y_h = C_1 e^{2t} + C_2 e^{-2t}$

$y_p = at + b$

$y'_p = a$ ، $y'' = 0$

نعوض بالمعادلة:

~~$y''_t - y'_t + y'_t - 4y = \ln x$~~

~~$y''_t - 4y = \ln e^t$~~

~~m^2~~

$0 - 4(at + b) = t$

$-4a = 1 \Rightarrow a = -\frac{1}{4}$

$b = 0$

$y = y_p + y_h$

$y = -\frac{1}{4}t + C_1 e^{2t} + C_2 e^{-2t}$

$y = -\frac{1}{4}\ln x + C_1 x^2 + \frac{C_2}{x^2}$

③ $(2x-3)^2 y'' - 10(2x-3)y' + 36y = 12(2x-3)^2$

نفرض $Z = 2x-3 \Leftarrow$

$y'_x = \frac{dy}{dx} = \frac{dy}{dZ} \cdot \frac{dZ}{dx}$

$y'_x = 2y'_Z \ , \ y'' = 4y''_Z$

$4Z^2 y'' - 20 y'_Z + 36y = 12Z^2$

$Z^2 y''_Z - 5Z y'_Z + 9y = 3Z^2$

نفرض $Z = e^t \Leftarrow$

$Z y'_Z = y'_t \ , \ Z^2 y''_Z = y''_t - y'_t$

$y''_t - 6y'_t + 9y = 3e^{2t}$

$m^2 - 6m + 9 = 0$

$(m-3)^2 = 0 \Rightarrow m_1 = m_2 = 3 \Rightarrow y_h = (C_1 + C_2 t)e^{3t}$

$y_p = \frac{1}{(D-2)^2} 3e^{2t} = 3Z^2$

$y = y_h + y_p$

$y = (C_1 + C_2 \ln Z)Z^3 + 3Z^2$

$y = (C_1 + C_2 \ln(2x-3))(2x-3)^3 + 3(2x-3)^2$

④ $3xy'' + 6x^2 y' + 3x^3 y = 3x \cdot e^{x - \frac{x^2}{2}}$

نقسم على $3x \Leftarrow$

$y'' + 2xy' + x^2 y = e^{x - \frac{x^2}{2}}$

$-\frac{P'}{2} - \frac{P^2}{4} + q = -1 - x^2 + x^2 = -1$ (الشرط محقق)

$y = u \cdot v \Rightarrow u = e^{-\int \frac{P(x)}{2} dx} = e^{-\frac{x^2}{2}}$

$y = e^{-\frac{x^2}{2}} \cdot V$

$y' = -x e^{-\frac{x^2}{2}} V + \cancel{x} \cdot e^{-\frac{x^2}{2}} V'$

$y'' = -e^{-\frac{x^2}{2}} V + x^2 e^{-\frac{x^2}{2}} V - V' x e^{-\frac{x^2}{2}} + V'' e^{-\frac{x^2}{2}} - x \cdot e^{-\frac{x^2}{2}} V'$

نعوض في المعادلة نجد:

$V'' - V = e^x$

$m^2 - 1 = 0$

$m_1 = -1 , m_2 = 1$

$V_h = C_1 \cdot e^{-x} + C_2 \cdot e^x$

$V_p = Ax \cdot e^x$

$V'_p = Ae^x + Axe^x$

$V''_p = 2Ae^x + Axe^x$

بالتعويض:

$2Ae^x = e^x \Rightarrow A = \frac{1}{2}$

$V_p = \frac{1}{2} x e^x$

$V = V_p + V_h$

$V = C_1 e^x + C_2 e^{-x} + \frac{1}{2} x e^x$

$y = (C_1 \cdot e^x + C_2 \cdot e^{-x} + \frac{1}{2} x e^x)^{-\frac{x^2}{2}}$

5. $x^2 y'' = y'^2$

خالية من y

$y' = Z , y'' = Z' \Rightarrow x^2 Z' = Z^2$

$x^2 \frac{dZ}{dx} = Z^2 \Rightarrow \frac{dZ}{Z^2} = \frac{dx}{x^2} \Rightarrow -\frac{1}{Z} = -\frac{1}{x} + C$

$$-\frac{1}{Z} = \frac{-1+xc}{x}$$

$$Z = \frac{x}{1-cx} \Longrightarrow y' = \frac{x}{1-cx}$$

$$y' = -\frac{1}{c}\cdot\frac{x}{x-\frac{1}{c}} = -\frac{1}{c}\cdot\frac{x-\frac{1}{c}+\frac{1}{c}}{x-\frac{1}{c}} = -\frac{1}{c}\left[x+\frac{1}{c}\cdot\frac{1}{x-\frac{1}{c}}\right]$$

$$y = -\frac{1}{c}\left[x+\frac{1}{c}\ln\left(\frac{1}{x-\frac{1}{c}}\right)+c\right].$$

(6): $9x^2y''+9xy'+(x^2-1)y=0$

نقسم على امثال $y'' \Longleftarrow$

$$y''+\frac{1}{x}y'+\left(\frac{1}{9}-\frac{1}{9x^2}\right)y=0$$

$$-\frac{p'}{2}+\frac{p^2}{2}+q = \frac{1}{2x^2}-\frac{1}{4x^2}+\frac{1}{9}-\frac{1}{9x^2}=\frac{1}{9}\ (\text{ثابت})$$

$$y=u.v \Rightarrow u = e^{-\int\frac{P(x)}{2}dx} = e^{-\frac{1}{2}\int\frac{1}{x}dx}$$

$$u=\frac{1}{\sqrt{x}},\quad y'=\frac{1}{2x\sqrt{x}}v+v'\cdot\frac{1}{\sqrt{x}}$$

$$y''=\frac{2\sqrt{x}+\sqrt{x}}{4x^3}\cdot v - v'\frac{1}{2x\sqrt{x}}+\frac{v''}{\sqrt{x}}-\frac{1}{2x\sqrt{x}}v'$$

بالتعويض بالمعادلة نجد أنه:

$$v''+\frac{1}{4}v=0$$

$$m^2+\frac{1}{4}=0$$

$$v=c_1\cos\frac{1}{2}x+c_2\sin\frac{1}{2}x.$$

$$y=u.v=\frac{1}{\sqrt{x}}c_1\cos\frac{1}{2}x+\frac{1}{\sqrt{x}}c_2\sin\frac{1}{2}x.$$

(7) $x^2y'' - xy' - 3y = x^5$

$x^2y'' = y''_t - y'_t \ , \ xy' = y'_t \ , \ x = e^t$

$y''_t - 2y'_t - 3y = e^{5t}$ نعوض بالمعادلة

$m^2 - 2m - 3 = 0$

$y_h = c_1 . e^{-x} + c_2 \, e^{3x}$

نفرض:

$y_p = A . e^{5t}$

$y'_p = 5Ae^{5t}$

$y''_p = 25\,Ae^{5t}$

بالتعويض نجد: $A = \frac{1}{12} \Leftarrow$

$y_p = \frac{1}{12} e^{5t}$

$\Longrightarrow y = y_h + y_p$

$y = c_1 . e^{-x} + c_2 . e^{3x} + \frac{1}{12} e^{5t}$

$y = c_1 . e^{-t} + c_2 . e^{3t} + \frac{1}{12} e^{5t}$

$y = c_1 \frac{1}{x} + c_2 x^3 + \frac{1}{12} x^5$

انتهت المحاضرة