كلية العلوم

القسم : **الرياضيات**

السنة : **الثالثة**

المادة : معادلات تفاضلية جزئية

المحاضرة : التاسعة/عملي/

{{ مكتبة A to Z }}

Facebook Group : مكتبة A to Z

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر (What's app-Telegram) على الرقم 0931497960

القسم : رياضيات

السنة : الثالثة

المادة : معادلات تفاضلية جزئية

المحاضرة : ك عملي والأخيرة

(1) $U_t = U_{xx}$ $\quad U(0,t)=1$ & $U(x,0)=0$

$x>0$ & $t>0$

نجد $U(x,t)$

$$S\,U(x,s) - U(x,0) = U_{xx}(x,s)$$

$$SU(x,s) = U_{xx}(x,s) \Rightarrow$$

$$U_{xx}(x,s) - SU(x,s) = 0 \Rightarrow$$

$$m^2 - s = 0 \Rightarrow m = \mp\sqrt{s}$$

$$\Rightarrow U(x,s) = C_1 e^{\sqrt{s}x} + C_2 e^{-\sqrt{s}x}$$

ولكن $U(x,s)$ تؤول $\Rightarrow$ $\boxed{C_1 = 0}$ $\Rightarrow$

$$U(x,s) = C_2 e^{-\sqrt{s}x}$$

$$U(0,s) = \frac{1}{s} \Rightarrow \boxed{C_2 = \frac{1}{s}}$$

$$\Rightarrow U(x,s) = \frac{e^{-\sqrt{s}x}}{s}$$

$$U(x,t) = \mathcal{L}^{-1}(U(x,s)) = \mathcal{L}^{-1}\left(\frac{e^{-\sqrt{s}x}}{s}\right) \Rightarrow$$

$$\boxed{U(x,t) = erf\ \frac{x}{2\sqrt{t}}}$$

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(2) $ty'' - y' + 4ty = 0$

$y(0) = 8$ & $y'(0) = 0$

$$\frac{d}{ds}\left(s^2Y(s) - sy(0) - y'(0)\right) - sY(s) + y(0) + 4\frac{d}{ds}\left(Y(s)\right) = 0$$

$$2sY(s) + s^2Y'(s) - 8 - sY(s) + 8 + 4Y'(s) = 0$$

$$(s^2+4)Y'(s) + sY(s) = 0$$

$$\Rightarrow \frac{Y'(s)}{Y(s)} = \frac{-s}{s^2+4}$$

$$Y(s) = \frac{c_1}{\sqrt{s^2+4}} \Rightarrow y(t) = \mathcal{L}^{-1}\left(Y(s)\right) = \mathcal{L}^{-1}\left(\frac{c_1}{\sqrt{s^2+4}}\right)$$

$$y(t) = c_1 J_0(2t) \Rightarrow y(0) = c_1 J_0(0)$$

$$\boxed{c_1 = 8} \Rightarrow \boxed{y(t) = 8J_0(2t)}$$

(3) $U_x + U_t = x$ — $u(0,t) = u(x,0) = 0$

$x > 0,\ t > 0$

$$U_x(x,s) + sU(x,s) - u(x,0) = \frac{x}{s}$$

$$U_x(x,s) + sU(x,s) = \frac{x}{s}$$

$$\mu = e^{sx} \Rightarrow U(x,s) = e^{-sx}\left[c_1 + \int \frac{x}{s} e^{sx}\,dx\right]$$

$$= e^{-sx}\left[c_1 + \frac{x}{s}\cdot\frac{e^{sx}}{s} - \frac{1}{s}\int \frac{e^{sx}}{s}\,dx\right]$$

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$$U(x,s) = e^{-sx}\left[c_1 + \frac{x}{s^2}e^{sx} - \frac{e^{sx}}{s^3}\right]$$

$$U(x,s) = c_1 e^{-sx} + \frac{x}{s^2} - \frac{1}{s^3}$$

$$U(0,s) = 0 \Rightarrow c_1 - \frac{1}{s^3} = 0 \Rightarrow \boxed{c_1 = \frac{1}{s^3}}$$

$$U(x,s) = \frac{e^{-sx}}{s^3} + \frac{x}{s^2} - \frac{1}{s^3}$$

$$u(x,t) = \mathcal{L}^{-1}(U(x,s)) = \mathcal{L}^{-1}\left[\frac{e^{-sx}}{s^3} + \frac{x}{s^2} - \frac{1}{s^3}\right]$$

$$u(x,t) = u(t-x)\frac{(t-x)^2}{2} + xt - \frac{t^2}{2}$$

(4) $y'' + 4\pi^2 y = \delta(t-2)$, $y(0)=0$, $y'(0)=2$

$$s^2Y(s) - sy(0) - y'(0) + 4\pi^2 Y(s) = e^{-2s}$$

$$(s^2 + 4\pi^2)Y(s) = e^{-2s} + 2$$

$$Y(s) = \frac{e^{-2s}}{s^2+4\pi^2} + \frac{2}{s^2+4\pi^2}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left[\frac{e^{-2s}}{s^2+4\pi^2} + \frac{2}{s^2+4\pi^2}\right]$$

$$y(t) = u(t-2)\frac{\sin(2\pi(t-2))}{2\pi} + \frac{\sin(2\pi t)}{\pi}$$

انتهت المحاضرة

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