كلية العلوم

القسم : **الرياضيات**

السنة : **الثالثة**

المادة : معادلات تفاضلية جزئية

المحاضرة : الثامنة/عملي/

{{ A to Z مكتبة }}

Facebook Group : A to Z مكتبة

القسم : رياضيات

السنة : الثالثة — المحاضرة : الثامنة عملي

المادة : معادلات تفاضلية جزئية

السؤال الأول :

⑤ $\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+1}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$

$\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = 1 - e^{-x}$

طريقة ثانية :

$\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) * \mathcal{L}^{-1}\left(\frac{1}{s+1}\right) = 1 * e^{-x}$

$= \int_0^x 1\, e^{-m}\, dm = \left[-e^{-m}\right]_0^x = 1 - e^{-x}$

⑥ $\mathcal{L}^{-1}\left(\frac{1}{(s-1)(s-2)}\right) = \mathcal{L}^{-1}\left(\frac{-1}{s-1} + \frac{1}{s-2}\right)$

$= \mathcal{L}^{-1}\left(\frac{1}{s-2}\right) - \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = e^{2x} - e^{x}$

طريقة ثانية :

$\mathcal{L}^{-1}\left(\frac{1}{(s-1)(s-2)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) * \mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$

$= e^{x} * e^{2x} = \int_0^x e^{x-m} \cdot e^{2m}\, dm = \int_0^x e^{x+m}\, dm =$

$= \left[e^{x+m}\right]_0^x = e^{2x} - e^{x}$

(7) $\mathcal{L}^{-1}\left(\frac{1}{s^4}\right) = \frac{1}{3!}\mathcal{L}^{-1}\left(\frac{3!}{s^4}\right) = \frac{x^3}{6}$

(8) $\mathcal{L}^{-1}\left(\frac{4s+12}{s^2+8s+16}\right) = \mathcal{L}^{-1}\left(\frac{4s+12}{(s+4)^2}\right)$

$= \mathcal{L}^{-1}\left(\frac{-4}{(s+4)^2} + \frac{4}{(s+4)}\right) = -4\mathcal{L}^{-1}\left(\frac{1}{(s+4)^2}\right) + 4\mathcal{L}^{-1}\left(\frac{1}{s+4}\right)$

$= -4te^{-4t} + 4e^{-4t}$

(9) $\mathcal{L}^{-1}\left(\frac{s-2}{s^2-4s+13}\right) = \mathcal{L}^{-1}\left(\frac{s-2}{(s-2)^2+9}\right) \Rightarrow e^{2x}\cos 3x$

(10) $\mathcal{L}^{-1}\left(\frac{6}{(s-1)^4}\right) = x^3 e^x$

(11) $\mathcal{L}^{-1}\left(\frac{4s}{(s^2+4)^2}\right) = 2\mathcal{L}^{-1}\left(\frac{2s}{(s^2+4)^2}\right) = 2x\sin 2x$

السؤال الثاني :

أوجد حل المعادلات التفاضلية الآتية باستخدام تحويل لابلاس :

(1) $y'' - 3y' + 2y = 4e^{2x}$; $y(0) = -3$
$y'(0) = 5$

$s^2Y(s) - sy(0) - y'(0) - 3sY(s) + 3y(0) + 2Y(s) = \frac{4}{s-2}$

$(s^2 - 3s + 2)Y(s) = \frac{4}{s-2} - 3s + 14$

$Y(s) = \frac{4}{(s-2)^2(s-1)} - \frac{3s-14}{(s-1)(s-2)}$

$$Y(s) = \frac{4}{(s-2)^2} - \frac{4}{(s-2)} + \frac{4}{(s-1)} - \frac{11}{(s-1)} + \frac{8}{(s-2)}$$

$$= \frac{4}{(s-2)^2} + \frac{4}{(s-2)} - \frac{7}{(s-1)}$$

$$y(x) = 4x e^{2x} + 4e^{2x} - 7e^{x}$$

2

$$y''' - 3y'' + 3y' - y = x^2 e^x$$

$$y(0) = 1 \quad , \quad y'(0) = 0 \quad , \quad y''(0) = -2$$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - 3s^2 Y(s) - 3s y(0)$$
$$+ 3y'(0) + 3s Y(s) - 3y(0) - Y(s) = \frac{2}{(s-1)^3}$$

$$(s^3 - 3s^2 + 3s - 1) Y(s) - s^2 + 2 + 3s - 3 = \frac{2}{(s-1)^3}$$

$$(s-1)^3 Y(s) = \frac{2}{(s-1)^3} + s^2 - 3s + 1$$

$$Y(s) = \frac{2}{(s-1)^6} + \frac{s^2 - 3s + 1}{(s-1)^3} = \frac{2}{(s-1)^6} + \frac{s^2 - 2s + 1 - s}{(s-1)^3}$$

$$= \frac{2}{(s-1)^6} + \frac{1}{(s-1)} - \frac{s}{(s-1)^3} = \frac{2}{(s-1)^6} + \frac{1}{(s-1)} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3}$$

$$= \frac{2}{(s-1)^6} + \frac{1}{(s-1)} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3}$$

$$y(x) = \frac{1}{60} x^5 e^x + e^x - x e^x - \frac{1}{2} x^2 e^x$$

(3) $y'' + 2y' + 5y = e^{-t}\sin t$

$y(0) = y'(0) = 0$

$s^2 Y(s) - s\,y(0) - y'(0) + 2s\,Y(s) - 2y(0) + 5Y(s) = \dfrac{1}{(s+1)^2+1}$

$(s^2+2s+5)Y(s) = \dfrac{1}{(s+1)^2+1}$

$Y(s) = \dfrac{1}{[(s+1)^2+1][(s+1)^2+4]} = \dfrac{\frac{1}{3}}{(s+1)^2+1} - \dfrac{\frac{1}{3}}{(s+1)^2+4}$

$y(x) = \frac{1}{3} e^{-x}\sin x - \frac{1}{3} e^{-x}\sin 2x$

(4) $u_x + u_t = x + t$ $\qquad u(0,t) = 1$

$u(x,0) = 1$

$U_x(x,s) + sU(x,s) - u(x,0) = \dfrac{x}{s} + \dfrac{1}{s^2}$

$U_x(x,s) + sU(x,s) = \dfrac{x}{s} + \dfrac{1}{s^2} + 1$

$\mu = e^{sx}$

$U(x,s) = e^{-sx}\left[c_1 + \int e^{sx}\left[\frac{x}{s} + \frac{1}{s^2} + 1\right]dx\right]$

$U(x,s) = e^{-sx}\left[c_1 + \left[\frac{x}{s} + \frac{1}{s^2} + 1\right]\frac{e^{sx}}{s} - \int \frac{e^{sx}}{s^2}dx\right]$

$U(x,s) = e^{-sx}\left[c_1 + \left[\frac{x}{s} + \frac{1}{s^2} + 1\right]\frac{e^{sx}}{s} - \frac{e^{sx}}{s^3}\right]$

$$U(x,s)=C_1e^{-sx}+\frac{x}{s^2}+\frac{1}{s}+\frac{1}{s^3}-\frac{1}{s^3}$$

$$=C_1e^{-sx}+\frac{x}{s^2}+\frac{1}{s}$$

$$u(0,t)=1 \Rightarrow U(0,s)=\frac{1}{s}$$

$$U(0,s)=C_1+\frac{1}{s}=\frac{1}{s} \Rightarrow \boxed{C_1=0}$$

$$U(x,s)=\frac{x}{s^2}+\frac{1}{s}$$

$$u(x,t)=tx+1$$

5) $u_{tt}=u_{xx}+\sin\pi x \quad ; \quad t>0$, $0<x<1$

$$u(x,0)=u(0,t)=u(1,t)=u_t(x,0)=0$$

$$s^2U(x,s)-sU(x,0)-u_x(x,0)=U_{xx}(x,s)+\frac{\sin\pi x}{s}$$

$$U_{xx}(x,s)-s^2U(x,s)=\frac{-\sin\pi x}{s}$$

$$m^2-s^2=0 \Rightarrow m=\mp s$$

$$U_h(x,s)=c_1e^{sx}+c_2e^{-sx}$$

$$U_p(x,s)=\frac{1}{D^2-s^2}\left(\frac{-\sin\pi x}{s}\right)=\frac{\sin\pi x}{s(s^2+\pi^2)}$$

$$U(x,s) = C_1 e^{sx} + C_2 e^{-sx} + \frac{\sin \pi x}{s(s^2+\pi^2)}$$

$$U(0,s) = U(1,s) = 0$$

$$U(0,s) = C_1 + C_2 = 0 \Rightarrow \boxed{C_1 = -C_2}$$

$$U(1,s) = C_1 (e^{s} - e^{-s}) = 0 \Rightarrow \boxed{C_1 = C_2 = 0}$$

$$\cancel{U_s(1,s) = C_1 (e^{s} - e^{-s}) = 0 \Rightarrow C}$$

$$U(x,s) = \frac{\sin \pi x}{s(s^2+\pi^2)} = \sin \pi x \left(\frac{1}{\pi^2 s} - \frac{s}{\pi^2 (s^2+\pi^2)} \right)$$

$$U(x,s) = \frac{\sin \pi x}{\pi^2} \left(\frac{1}{s} - \frac{s}{s^2+\pi^2} \right)$$

$$u(x,t) = \frac{\sin \pi x}{\pi^2} (1 - \cos \pi t)$$

انتهت المحاضرة