



كلية العلوم

القسم : الرياضيات

السنة : الثانية

المادة : معادلات تفاضلية ١

المحاضرة : التاسعة / عملي /

{{ مكتبة A to Z }}

مكتبة A to Z : Facebook Group

كلية العلوم ، كلية الصيدلة ، الهندسة التقنية

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يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر (What's app-Telegram) على الرقم 0931497960

الدكتور:

المحاضرة:

عملية



التاريخ: / /

القسم: الرياضيات

السنة: الثانية

المادة: معادلات تفاضلية 1

A to Z Library for university services

$$\textcircled{1} y'' + y' - 2y = x^2 + 3.$$

$$m^2 + m - 2 = 0$$

الحل:

$$(m+2)(m-1)=0 \Rightarrow m=-2, m=1$$

$$y_h = C_1 e^{-2x} + C_2 e^x$$

$$y_p = x^2 A_2 + x A_1 + A_0 \quad \text{نضرب بـ } x-2 \quad \begin{matrix} -2 \\ 1 \end{matrix}$$

$$y_p' = 2x A_2 + A_1 \quad \text{نضرب بـ } x-1 \quad \begin{matrix} 1 \\ 1 \end{matrix}$$

$$y_p'' = 2A_2 \quad \text{نضرب بـ } x-1 \quad \begin{matrix} 1 \\ 1 \end{matrix}$$

بالجمع بعد الضرب نجد:

$$-2x^2 A_2 + (-2A_1 + 2A_2)x - 2A_0 + A_1 + 2A_2 = 2x^2 + 3$$

$$-2A_2 = 2 \Rightarrow A_2 = -1$$

$$-2A_1 + 2A_2 = 0$$

$$A_1 = A_2 = -1$$

$$-2A_0 + A_1 + 2A_2 = 3$$

$$-2A_0 = 6 \Rightarrow A_0 = -3$$

$$y_p = -x^2 - x - 3$$

$$y = y_p + y_h = C_1 e^{-2x} + C_2 e^x - x^2 - x - 3$$

$$\textcircled{2} y^{(4)} - 16y'' = e^{2x}$$

$$\text{نضرب بـ } x^2 - 2x + 2$$

$$m^4 - 16m^2 = 0 \Rightarrow m^2(m^2 - 16) = 0$$

$$m_1 = m_2 = 0, m_3 = +4, m_4 = -4$$

$$(C_1 + C_2x) + C_3e^{4x} + C_4e^{-4x}$$

$$y_p = A.e^{2x}$$

$$y'_p = 2Ae^{2x}$$

$$y''_p = 4Ae^{2x} \quad \text{---} \quad x(-16) \quad \text{---} \quad (1)$$

$$y'''_p = 8Ae^{2x}$$

$$y''''_p = 16Ae^{2x} \quad \text{---} \quad x(1) \quad \text{---} \quad (2)$$

نأخذ y'''' و y'' لنعوض

$$-48Ae^{2x} = e^{2x}$$

$$A = -\frac{1}{48} \Rightarrow y_p = -\frac{1}{48}e^{2x} \Rightarrow y = y_h + y_p$$

$$(3) \quad y'' + 9y = 12x^2e^{3x} + 6$$

$$m^2 + 9 = 0 \Rightarrow m = \pm 3i$$

$$y_h = C_1 \cos 3x + C_2 \sin 3x$$

$$y_p = (ax^2 + bx + c)e^{3x} + d$$

$$y'_p = (2ax + b)e^{3x} + 3e^{3x}(ax^2 + bx + c)$$

$$y''_p = 2ae^{3x} + 3e^{3x}(2ax + b) + 9e^{3x}(ax^2 + bx + c) + (2x + b)3e^{3x} \\ = e^{3x}(18a^2 + (18b + 12a)x + (18c + 2a + 4b)) = 12x^2e^{3x} + 6$$

$$18a = 12 \Rightarrow a = \frac{2}{3}$$

$$18b + 12a = 0 \Rightarrow b = -\frac{12}{18} \Rightarrow b = -\frac{4}{6} = -\frac{2}{3}$$

$$18c + 2a + 4b = 0 \Rightarrow c = -\frac{2a + 4b}{18} = -\frac{1}{18}\left(\frac{4}{3} - \frac{8}{3}\right)$$

$$c = \frac{4}{54} = \frac{2}{17}$$

$$9d - 6 = 0 \Rightarrow d = \frac{2}{3}$$

$$y_p = \left(\frac{2}{3}x^2 - \frac{2}{3}x + \frac{2}{17} \right) e^{3x} + \frac{2}{3}$$

$$(4) \quad y'' - 3y' - 4y = e^{-x}$$

$$m^2 - 3m - 4 = 0$$

$$(m-4)(m+1) = 0$$

$$m=4, m=-1 \Rightarrow y_h = C_1 e^{4x} + C_2 e^{-x}$$

$$y_p = x A e^{-x} \quad (\text{not } 4x)$$

$$y'_p = A e^{-x} - A x e^{-x} \quad \text{--- } \times (-4)$$

$$y''_p = -A e^{-x} - A e^{-x} + A x e^{-x} \quad \text{--- } \times (-3)$$

بالضرب في 5

$$-5A e^{-x} = e^{-x} \Rightarrow -5A = 1 \Rightarrow A = -\frac{1}{5}$$

$$y_p = -\frac{1}{5} x e^{-x} \Rightarrow y = y_h + y_p$$

لحل
باستخدام الاغراض

$$y_h = C_1 e^{4x} + C_2 e^{-x}$$

$$y_p = C_1(x) e^{4x} + C_2(x) e^{-x}$$

$$y'_p = C'_1(x) e^{4x} + C'_2(x) e^{-x} + 4C_1(x) e^{4x} - C_2(x) e^{-x}$$

$$C'_1(x) e^{4x} + C'_2(x) e^{-x} = 0$$

$$y' = 4C_1(x) e^{4x} - C_2(x) e^{-x} \quad \text{--- } \times (-3)$$

$$y'' = 4C'_1(x) e^{4x} - C'_2(x) e^{-x} + 16C_1(x) e^{4x} + C_2(x) e^{-x} \quad \text{--- } \times (4)$$

$$5C'_1(x) e^{4x} = e^{-x}$$

$$C'_1(x) = \frac{1}{5} e^{-5x} \Rightarrow C_1(x) = -\frac{1}{25} e^{-5x}$$

$$-5C_2'(x)e^{-x} = e^{-x}$$

$$-5C_2'(x) = 1 \Rightarrow C_2' = -\frac{1}{5}x$$

$$y_p = -\frac{1}{25}e^{-x} - \frac{1}{5}xe^{-x}$$

$$(5) \quad y'' + 2y' + y = e^x \cos x$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0 \Rightarrow m_1 = m_2 = -1 \Rightarrow y_h = (C_1 + C_2x)e^{-x}$$

$$y_p = (A \cos x + B \sin x)e^x$$

$$y_p' = (-A \sin x + B \cos x)e^x + e^x(A \cos x + B \sin x) \quad \text{--- (1)}$$

$$y_p'' = (-A \sin x + B \cos x - A \sin x + B \cos x)e^x \quad \text{--- (2)}$$

$$y_p'' = (-2A \sin x + 2B \cos x)e^x$$

$$y_p'' = (-2A \sin x + 2B \cos x)e^x$$

$$y_p'' = (-2A \sin x + 2B \cos x)e^x \quad \text{--- (1)}$$

$$((3A + 4B) \cos x + (-4A + 3B) \sin x)e^x = e^x \cos x$$

$$3A + 4B = 1$$

$$-4A + 3B = 0$$

بالحل المتكامل نجد:

$$A = \frac{3}{25}, \quad B = \frac{4}{25} \Rightarrow y_p = \left(\frac{3}{25} \cos x + \frac{4}{25} \sin x \right) e^x$$

$$y = y_h + y_p$$

$$(6) \quad y^{(4)} + y'' = \cos x$$

$$m^4 + m^2 = 0 \Rightarrow y_h = C_1 + C_2x + C_3 \sin x + C_4 \cos x$$

$$y_p = x(A \cos x + B \sin x)$$

$$y'_p = (A \cos x + B \sin x) + x(-A \sin x + B \cos x)$$

$$y''_p = -2A \sin x + 2B \cos x + x(-A \cos x - B \sin x)$$

$$y'''_p = (-2A - B) \sin x + (2B - A) \cos x + x(A \sin x - B \cos x)$$

$$y''''_p = (A \cos x + B \sin x)x + 4A \sin x - 4B \cos x$$

$$2A \sin x - 2B \cos x = \cos x$$

$$A = 0, B = -\frac{1}{2} \Rightarrow y_p = -\frac{1}{2} x \sin x \Rightarrow y = y_h + y_p$$

$$\textcircled{7} \cdot y''' - 3y'' + 3y' - y = e^x$$

$$m^3 - 3m^2 + 3m - 1 = 0$$

$$(m-1)^3 = 0 \Rightarrow m_1 = m_2 = m_3 = 1 \Rightarrow y_h = C_1 + C_2 x + C_3 x^2$$

$$y_p = A x^3 e^x \quad \text{--- } x(-1)$$

$$y'_p = 3A x^2 e^x + A x^3 e^x \quad \text{--- } x(3)$$

$$y''_p = 6A x e^x + 6A x^2 e^x + A x^3 e^x \quad \text{--- } x(-3)$$

$$y'''_p = 6A e^x + 18A x e^x + 9A x^2 e^x + A x^3 e^x \quad \text{--- } (1)$$

بالجمع والضرب نجد:

$$6A e^x - e^x \Rightarrow A = \frac{1}{6}$$

$$y = y_p + y_h$$

انتهى الحل