كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : معادلات تفاضلية١

المحاضرة : الثامنة/عملي/

{{ مكتبة A to Z }}

Facebook Group : مكتبة A to Z

① $y''' - 3y'' + 4y = 2e^x$

$m^3 - 3m^2 + 4 = 0 \quad ; m = -1 \Rightarrow$ نقسم على $m+1$

$$\begin{array}{r|l} m+1 & m^3 - 3m^2 + 4 \\ & m^3 + m^2 \\ \hline & -4m^2 + 4 \\ & -4m^2 - 4m \\ \hline & 4m + 4 \\ & 4m + 4 \\ \hline & 0 \end{array} \qquad m^2 - 4m + 4$$

$\Rightarrow (m+1)(m^2 - 4m + 4) = 0$

$m + 1 = 0 \Rightarrow m = -1$

$m = -2$ مكرر مضاعف

$y_h = C_1 e^{-x} + (C_2 + C_3 x) e^{2x}$

$y_p = \frac{1}{(D+1)(D-2)^2} 2e^x = \frac{1}{(1+1)(1-2)^2} \cdot 2e^x = e^x$

$y = y_h + y_p$

② $y''' - 3y'' + 4y = e^{2x}$

$y_h = C_1 e^{-x} + (C_2 + C_3 x) e^{2x}$

$y_p = \frac{1}{(D+1)(D-2)^2} e^{2x} = \frac{1}{(D-2)^2} \cdot \frac{1}{D+1} e^{2x} = \frac{1}{(D-2)^2} \cdot \frac{1}{3} e^{2x}$

$= \frac{x^2}{2} \cdot \frac{1}{3} e^{2x} = \frac{x^2 \cdot e^{2x}}{6}$

$y = y_h + y_p$

(3) $y'' + y = \cos 3x$

$m^2 + 1 = 0$

$m^2 = -1 \Rightarrow m_1 = m_2 = \pm i$

$y_h = C_1 \cos x + C_2 \sin x$

$a^2 = -(3)^2 = -9$

$y_p = \frac{1}{D^2+1} \cos 3x$

$y_p = \frac{1}{-9+1} \cos 3x = -\frac{1}{8} \cos 3x$

$y = y_h + y_p$

(4) $y'' + 2y' = \cos x$

$m^2 + 2m = 0 \Rightarrow m(m+2) = 0 \Rightarrow m = 0, m = -2$

$y_h = C_1 + C_2 e^{-2x}$

$y_p = \frac{1}{(D^2+2D)} \cos x = \frac{1}{D(D+2)} \cos x = \frac{1}{D}\left(\frac{1}{D+2} \cos x\right)$

$= \frac{1}{D}\left(\frac{D-2}{(D+2)(D-2)} \cdot \cos x\right)$

$= \frac{1}{D}\left((D-2) \cdot \frac{1}{D^2-4}\right) \cos x = \frac{1}{D}(D-2)\left(-\frac{1}{5} \cos x\right)$

$= \frac{1}{5}\left(\frac{1}{D}\right)(-\sin x - 2\cos x) = -\frac{1}{5}(\cos x - 2\sin x) \Rightarrow y = y_h + y_p$

(5) $y'' - 2y' + y = x e^x$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = +1$ مكرر

$y_h = (C_1 + C_2 x) e^x$

$y_p = \frac{1}{(D^2-1)^2} \cdot x e^x = \frac{1}{(D+1-1)^2} x e^x = e^x \frac{1}{D^2} x$

$= e^x \cdot \frac{x^3}{6}$

$y = y_p + y_h$

(6) $y'' + 4y' + 3y = x^2 - 3x$

$m^2 + 4m + 3 = 0$

$(m+3)(m+1) = 0$

$m = -3, \; m = -1$

$y_h = C_1 e^{-3x} + C_2 e^{-x}$

$y_p = \frac{1}{(D^2+4D+3)} (x^2 - 3x)$

$$
\begin{array}{r|l}
D^2+4D+3 & 1 \\
\hline
 & \frac{1}{3} - \frac{4}{9}D + \frac{13}{27}D^2 \\
 & 1 - \frac{4}{3}D + \frac{1}{3}D^2 \\
\hline
 & -\frac{4}{3}D - \frac{1}{3}D^2 \\
 & -\frac{4}{3}D + \frac{16}{9}D^2 - \frac{4}{9}D^3 \\
\hline
 & \frac{13}{9}D^2 + \frac{4}{9}D^3
\end{array}
$$

$$y_p = \left(\frac{1}{3} - \frac{4}{9}D + \frac{13}{27}D^2\right)(x^2 - 3x)$$

$$= \frac{1}{3}x^2 - x - \frac{4}{9}(2x-3) + \frac{13}{27}(2)$$

$$= \frac{1}{3}x^2 - x - \frac{8x}{9} + \frac{12}{9} + \frac{26}{27}$$

$$y = y_h + y_p$$

(7) $y'' - y = 3e^{2x} - 2\cos x$

$$m^2 - 1 = 0$$

$$\Rightarrow m^2 = 1 \Rightarrow m = \mp 1$$

$$\Rightarrow y_h = C_1 e^x + C_2 e^{-x}$$

$$y_p = \frac{1}{D^2-1} 3e^{2x} - 2\cos x$$

$$= \frac{1}{D^2-1} 3e^{2x} - \frac{2}{D^2-1}\cos x$$

$$y_p = e^{2x} + \cos x$$

$$y = y_p + y_h$$

(8) $y'' + y = \cos x + 2\sin(2x)$

$$m^2 + 1 = 0 \Rightarrow m^2 = -1$$

$$\Rightarrow m_{1,2} = \mp i$$

$$\Rightarrow y_h = (C_1\cos x + C_2 \sin x)$$

$$y_p = \frac{1}{D^2+1}(\cos x + 2\sin 2x) = \frac{1}{D^2+1}\cos x + \frac{2}{D^2+1}\sin 2x$$

$$\Rightarrow y_p = \frac{1}{D^2+1}\cos x - \left(\frac{2}{3}\sin 2x\right)\cdot\frac{1}{D^2+1}\cos x - \frac{1}{D^2+1}\left(\frac{e^{ix} + e^{-ix}}{2}\right)$$

$$\frac{1}{D^2+1}\cdot\frac{e^{ix}}{2}+\frac{1}{D^2+1}\cdot\frac{e^{-ix}}{2}$$

$$=\frac{1}{(D+i)(D-i)}\frac{e^{ix}}{2}+\frac{1}{(D+i)(D-i)}\frac{e^{-ix}}{2}$$

$$=\frac{1}{D-i}\left(\frac{1}{D+i}\cdot\frac{e^{ix}}{2}\right)+\frac{1}{D+i}\left(\frac{1}{D-i}\cdot e^{-ix}\right)$$

$$=\frac{1}{D-i}\left(\frac{1}{x}\cdot\frac{e^{ix}}{2}\right)+\frac{1}{D+i}\left(-\frac{1}{2i}\cdot\frac{e^{ix}}{2}\right)$$

$$=x\left(\frac{1}{2i}\cdot\frac{e^{ix}}{2}\right)+x\left(-\frac{1}{2i}\cdot\frac{e^{-ix}}{2}\right)$$

$$=\frac{x}{2}\left[\frac{e^{ix}-e^{-ix}}{2i}\right]=\frac{x}{2}\sin x.$$

$$y_p=\frac{x}{2}\sin x-\frac{2}{3}\sin 2x$$

$$\Rightarrow y=y_h+y_p$$

انتهت المحاضرة