كلية العلوم

القسم : **الكيمياء**

السنة : **الاولى**

المادة : رياضيات عامة٢

المحاضرة :الثالثة/عملي/

{{ مكتبة A to Z }}

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(3) عملي

1. بين أنه إذا كان $y = \operatorname{argsh}[u(x)]$ فإن $y' = \dfrac{u'(x)}{\sqrt{1+u^2(x)}}$

ثم استنتج $\left(\operatorname{argsh}\left(\frac{x}{a}\right)\right)'$

الحل: نضع $f(x) = \operatorname{argsh} x \Rightarrow y = (f \circ u)(x)$

$$y' = f'(u(x)) \cdot u'(x) = \frac{1}{\sqrt{1+u^2(x)}} \times u'(x) = \frac{u'(x)}{\sqrt{1+u^2(x)}}$$

* $y = \operatorname{argsh}\left(\frac{x}{a}\right)$, $u(x) = \frac{x}{a} = \frac{1}{a}x \Rightarrow u'(x) = \frac{1}{a}$

$$\left(\operatorname{argsh}\left(\frac{x}{a}\right)\right)' = \frac{u'(x)}{\sqrt{1+u^2(x)}} = \frac{\frac{1}{a}}{\sqrt{1+\left(\frac{x}{a}\right)^2}} = \frac{\frac{1}{a}}{\sqrt{\frac{a^2+x^2}{a^2}}} = \frac{\frac{1}{a}}{\frac{\sqrt{x^2+a^2}}{a}}$$

$$\Rightarrow \left(\operatorname{argsh}\left(\frac{x}{a}\right)\right)' = \frac{1}{\sqrt{x^2+a^2}}$$

بالمثل نجد أن:

$$\left(\operatorname{argch}[u(x)]\right)' = \frac{u'(x)}{\sqrt{u^2(x)-1}} \Rightarrow \left(\operatorname{argch}\left(\frac{x}{a}\right)\right)' = \frac{1}{\sqrt{x^2-a^2}}$$

$$\left[\arcsin[u(x)]\right]' = \frac{u'(x)}{\sqrt{1-u^2(x)}} \Rightarrow \left(\arcsin\left(\frac{x}{a}\right)\right)' = \frac{1}{\sqrt{a^2-x^2}}$$

$$\left[\arccos(u(x))\right]' = \frac{-u'(x)}{\sqrt{1-u^2(x)}} \Rightarrow \left(\arccos\frac{x}{a}\right)' = \frac{-1}{\sqrt{a^2-x^2}}$$

$$\left[\operatorname{arctg} u(x)\right]' = \frac{u'(x)}{1+u^2(x)} \Rightarrow \left(\operatorname{arc\,tg}\frac{x}{a}\right)' = \frac{a}{x^2+a^2}$$

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2. أوجد المشتق لكل من الدوال التالية

1. $y = x - \sqrt{1-x^2} \cdot \arcsin x$

$$y' = 1 - \left[\frac{-2x}{2\sqrt{1-x^2}} \cdot \arcsin x + \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} \right]$$

$$= 1 + \frac{x \cdot \arcsin x}{\sqrt{1-x^2}} - 1 = \frac{x \cdot \arcsin x}{\sqrt{1-x^2}}$$

2. $y = \operatorname{arctg}(3x) + \operatorname{argsh}(2x+1)$

$$y' = \frac{(3x)'}{1+(3x)^2} + \frac{(2x+1)'}{\sqrt{(2x+1)^2+1}} = \frac{3}{1+9x^2} + \frac{2}{\sqrt{(2x+1)^2+1}}$$

3. $y = \arccos\left(\frac{x}{7}\right) + tg^2(5x)$

$$y' = \frac{-1}{\sqrt{49-x^2}} + 2\, tg(5x) \times 5 \left(1 + tg^2(5x)\right)$$

$$= -\frac{1}{\sqrt{49-x^2}} + 10\, tg(5x) + 10\, tg^3(5x)$$

4. $y = a^x$

$$y = e^{x \cdot \ln a} \Rightarrow y' = \ln(a) \cdot e^{x \ln a}$$

$$y' = \ln(a)\, a^x$$

③

3. أوجد تفاضل الدالة $y = \ln\left[\operatorname{tg}\left(\frac{x}{\sqrt{1-x^2}}\right)\right]$

نعلم أن تفاضل الدالة هو $dy = y' \cdot dx$

نضع $U(x) = \frac{x}{\sqrt{1-x^2}}$ فيكون :

$$U'(x) = \frac{1 \cdot \sqrt{1-x^2} - \frac{-x}{\sqrt{1-x^2}} \cdot x}{1-x^2} = \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{1}{(1-x^2)\sqrt{1-x^2}} = \frac{1}{(1-x^2)^{\frac{3}{2}}}$$

$$dy = \frac{\left[\operatorname{tg}\left(\frac{x}{\sqrt{1-x^2}}\right)\right]'}{\operatorname{tg}\left(\frac{x}{\sqrt{1-x^2}}\right)} \cdot dx = \frac{(\operatorname{tg}[U(x)])'}{\operatorname{tg}(U(x))} \cdot dx$$

$$= \frac{u'(x)\left(1+\operatorname{tg}^2 U(x)\right)}{\operatorname{tg}(U(x))} dx = \frac{U'(x) \times \frac{1}{\cos^2 U(x)}}{\frac{\sin U(x)}{\cos U(x)}} \cdot dx$$

$$= \frac{U'(x)}{\sin U(x) \cdot \cos U(x)} = \frac{2\,U'(x)}{\sin 2U(x)}$$

$$dy = \frac{\frac{2}{(1-x^2)^{\frac{3}{2}}}}{\sin\left[\frac{2x}{\sqrt{1-x^2}}\right]} \cdot dx$$

$$dy = \frac{2\,dx}{(1-x^2)^{\frac{3}{2}} \cdot \sin\left(\frac{2x}{\sqrt{1-x^2}}\right)}$$

(4)

$$\left(\operatorname{arctg}\frac{x}{a}\right)' = \frac{a}{x^2+a^2}$$ توضيح

$$\left[\operatorname{arctg}\left(\frac{x}{a}\right)\right]' = \frac{\left(\frac{x}{a}\right)'}{1+\left(\frac{x}{a}\right)^2} = \frac{\frac{1}{a}}{1+\frac{x^2}{a^2}}$$

$$= \frac{\frac{1}{a}}{\frac{a^2+x^2}{a^2}} = \frac{1}{a} \times \frac{a^2}{x^2+a^2}$$

$$\left[\operatorname{arctg}\left(\frac{x}{a}\right)\right]' = \frac{a}{x^2+a^2}$$

$$\left(\arcsin\frac{x}{a}\right)' = \frac{1}{\sqrt{a^2-x^2}}$$

$$\left(\arcsin\frac{x}{a}\right)' = \frac{\left(\frac{x}{a}\right)'}{\sqrt{1-\left(\frac{x}{a}\right)^2}} = \frac{\frac{1}{a}}{\sqrt{1-\frac{x^2}{a^2}}} = \frac{\frac{1}{a}}{\frac{\sqrt{a^2-x^2}}{a^2}}$$

$$= \frac{\frac{1}{a}}{\frac{\sqrt{a^2-x^2}}{a}} = \frac{1}{\sqrt{a^2-x^2}}$$

$$\left(\arcsin\frac{x}{a}\right)' = \frac{1}{\sqrt{a^2-x^2}}$$