كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : معادلات تفاضلية١

المحاضرة : **الخامسة/عملي/**

{{ مكتبة A to Z }}

Facebook Group : مكتبة A to Z

2026

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر(What's app-Telegram)على الرقم 0931497960

تمرين: حل المعادلات التفاضلية:

1) $(x\ln x)y' + y = 2\ln(x)$

الحل: نقسم على $x\ln x$

$$y' + \frac{1}{x\ln x}y = \frac{2}{x}$$

نوجد الحل العام للمعادلة المتجانسة

$$y' + \frac{1}{x\ln x}y = 0$$

$$\frac{dy}{dx} + \frac{1}{x\ln x}y = 0 \Rightarrow dy + \frac{dx}{x\ln x}y = 0 \Rightarrow \frac{dy}{y} + \frac{dx}{x\ln x} = 0$$

$$\ln(y) + \ln(\ln x) = \ln(c)$$

$$y\ln(x) = c$$

نوجد الحل الخاص غير المتجانسة

$$y\ln(x) = c(x) \Rightarrow y = \frac{c(x)}{\ln(x)}$$

$$y' = \frac{c'\ln x - \frac{cx}{x}}{(\ln x)^2} = \frac{c'\ln x - \frac{c}{x}}{(\ln x)^2} + \frac{1}{\ln(x)}\left(\frac{c}{\ln x}\right) = \frac{2}{x}$$

$$c'\ln x - \frac{c}{x} + \frac{c}{x} = \frac{2}{x}\ln^2 x$$

$C' \ln x = \frac{2}{x} \ln^2 x$

$C' = \frac{2}{x} \ln x \Rightarrow C = \ln^2 x + C_1 \Rightarrow \boxed{y \ln x = \ln^2 x + C_1}$

$x^2 y - x^3 y' = y^4 \cos x$ ..... (2)

$x^2 y^{-3} - x^3 y' \cdot y^{-4} = \cos x$

$-x^3 y' y^{-4} + x^2 y^{-3} = \cos x$

$Z = y^{-3} \Rightarrow Z' = -3y^{-4} \cdot y'$

$\frac{1}{3} x^3 Z' + x^2 Z = \cos x$

نضرب بـ $\frac{Z}{x^3}$

$Z' + 3\frac{Z}{x} = \frac{3\cos x}{x^3}$

$\Rightarrow M = e^{\int \frac{3}{x} dx} = e^{3\ln(x)} = x^3$

$Z' x^3 + 3 Z x^2 = 3\cos x$

$x^3 dZ + 3x^2 Z dx = 3\cos dx$

$Z x^3 = 3\sin x dx + C$

$\frac{x^3}{y^3} = 3\sin x + C$ ..... (2)

(3) $(2x+y)dy = y dx + 4y^3 dy$

نقسم على dy

$(2x + y - 4y^3) - y\frac{dx}{dy} = 0$

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$-yx' + 2x = -y + 4y^3$

$x' - \frac{2x}{y} = 1 - 4y^2$

$\mu = e^{\int P(y)dy} = e^{\int -\frac{2}{y}dy} = \frac{1}{y^2}$

نضرب بـ $\frac{1}{y^2}$ ⟸

$\frac{1}{y^2}x' - \frac{2}{y^2}x = \left(\frac{1}{y^2} - 4\right)$

$\frac{1}{y^2}dx - \frac{2}{y^3}dy = \left(\frac{1}{y^2} - 4\right)dy$

$\frac{1}{y^2}x = -\frac{1}{y} - 4y + C$

$(\sin^2 y + x\cot y)y' = 1$ ..... (4)

$\sin^2 y = -x\cot y + x'$

$\mu = e^{\int P(y)dy} = e^{\int -\cot y dy} = e^{-\ln(\sin y)} = \frac{1}{\sin y}$

$\sin y = -x\frac{\cos y}{\sin^2 y} + \frac{x^2}{\sin y}$

$\sin y\,dy = -x\frac{\cos y}{\sin^2 y}dy + \frac{1}{\sin y}dx$

$-\cos y = \frac{1}{\sin y}x + C$

(5) $y' = y^4\cos x + y\tan x$

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$y' - y\tan x = y^4 \cos x$

نقسم على y^4

$y' \cdot y^{-4} - \tan x \cdot y^{-3} = \cos x$

نفرض $Z = y^{-3}$

$Z' = -3y^{-4} y' \Rightarrow \frac{Z'}{-3} = y^{-4} \cdot y'$

$\frac{Z'}{-3} - \tan x \, Z = \cos x$

$Z' + 3\tan x \, Z = -3\cos x$ ... خطية

$P(x) = 3\tan x \;,\; q(x) = -3\cos x$

$\mu = e^{\int P(x)\,dx} = e^{\int 3\tan x\,dx}$

$\mu = e^{3\int \frac{\sin x}{\cos x}} = e^{3\ln(\cos x)} \Rightarrow \mu = \frac{1}{\cos^3 x}$

نضرب طرفي المعادلة بـ μ

$\frac{1}{\cos^3 x} Z' + \frac{3\tan x}{\cos^3 x} Z = \frac{-3}{\cos^2 x}$

$\frac{1}{\cos^3 x} dZ + \frac{3\tan x \, Z}{\cos^3 x} dx + \frac{3}{\cos^2 x} dx = 0$

$\frac{3}{\cos^2 x} dx + \left(\frac{1}{\cos^3 x} dZ + \frac{3\tan x}{\cos^3 x} Z dx \right)$

$3\tan x + \frac{Z}{\cos^3 x} = c \Rightarrow$ ~~$3\tan x$~~

$3\tan x + \frac{y^{-3}}{\cos^3 x} = c$

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6. $(2y-x)dx - 2xdy + (y-x)x(xdy - ydx) = 0$

نفرض $z = \frac{y}{x}$

$y = x.z$

$dz = \frac{xdy - ydx}{x^2}$

$dy = xdz + zdx$

$(2xz - x)dx - 2x(xdz + zdx) + (zx - x)(x^2 dz) = 0$

نقسم على x

$((2z-1) - 2z)dx + (-2x + zx^2 - x^2)dz = 0$

$-dx + (2x + zx^2 - x^2)dz = 0$

$\frac{dx}{dz} - 2x = (1-z)x^2$

نقسم على x^2

$x^{-2}.x' + 2x^{-1} = (-1+z)$

نفرض $t = x^{-1}$ $t' = -x^{-2}x'$

$-t' + 2t = z - 1$

$(t' - 2t) = 1 - z$

$M = e^{\int -2dt} = e^{-2z}$

$e^{-2z}dt - 2e^{-2z}dz = (1-z)e^{-2z}dz$

نكامل

$e^{-2z} = (1-z)e^{-2z}$

بالمكاملة نجد أن

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$$e^{-2z}t = e^{-2z}\left(-\frac{3}{4}+\frac{1}{2}z\right)+C$$

$$e^{-\frac{2y}{x}}\cdot\frac{1}{x} = e^{-\frac{2y}{x}}\left(-\frac{3}{4}+\frac{1}{2}\frac{y}{x}\right)+C$$

(7) $y' = y^2 - \frac{2}{x^2}$; $y_1 = \frac{1}{x}$

$$y = y_1 + \frac{1}{z}$$

$$y = \frac{1}{x} + \frac{1}{z}$$

$$y' = -\frac{1}{x^2} - \frac{z'}{z^2}$$

$$-\frac{1}{x^2} - \frac{z'}{z^2} = \left(\frac{1}{x}+\frac{1}{z}\right)^2 - \frac{2}{x^2}$$

$$-\frac{1}{x^2} - \frac{z'}{z^2} = \frac{1}{x^2} + \frac{2}{xz} + \frac{1}{z^2} - \frac{2}{x^2}$$

$$-\frac{z'}{z^2} = \frac{2}{x\cdot z} + \frac{1}{z^2}$$

$-z^2$; نوعي $z' = -2\frac{z}{x} - 1$

$$z' + \frac{2z}{x} = -1 \rightarrow \mu = e^{\int \frac{2}{x}dx} = x^2$$

$$x^2\,dz + 2xz\,dx = -x^2\,dx$$

$$\rightarrow zx^2 = -\frac{x^3}{3}+C$$

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$$\frac{1}{z} = y - \frac{1}{x} \Rightarrow z = \frac{x}{xy-1}$$

$$\frac{x^3}{xy-1} = \frac{x^3}{3} + c$$

(8) $y' + 2y e^x - y^2 = e^{2x} + e^x$; $y = e^x$

$$y = y_1 + \frac{1}{z} \Rightarrow y = e^x + \frac{1}{z}$$

$$y' = e^x - \frac{z'}{z^2} \; ; \; y^2 = e^{2x} + \frac{2e^x}{z} + \frac{1}{z^2}$$

$$e^x - \frac{z'}{z^2} + 2e^x\left(e^x + \frac{1}{z}\right) - \left(e^{2x} + \frac{2e^x}{z} + \frac{1}{z^2}\right) = e^x + e^{2x}$$

$$e^x - \frac{z'}{z^2} + 2e^{2x} + \frac{2e^x}{z} - e^{2x} - \frac{2e^x}{z} - \frac{1}{z^2} = e^{2x} + e^x$$

$$-\frac{z'}{z} - \frac{1}{z^2} = 0$$

$$z' + 1 = 0$$

$$z' = -1 \Rightarrow z = -x + c$$

$$\frac{1}{z} = \frac{1}{-x+c}$$

$$\Rightarrow \boxed{y = e^x + \frac{1}{-x+c}}$$

انتهت المحاضرة

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