



كلية العلوم

القسم : الرياضيات

السنة : الثانية

المادة : معادلات تفاضلية ١

المحاضرة : الرابعة / عملي

{{ مكتبة A to Z }}

مكتبة A to Z : Facebook Group

كلية العلوم ، كلية الصيدلة ، الهندسة التقنية

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر (What's app-Telegram) على الرقم 0931497960

الدكتور:

المحاضرة:

..... 4 عملية



القسم: الرياضيات

السنة: الثانية

المادة: معادلات تفاضلية 1

التاريخ: / /

A to Z Library for university services

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تمرين: تحقق فيما إذا كانت المعادلات تامة ثم حلها

$$① (2xy^3 - 2x^3y^3 - 4xy^2 + 2x)dx + (3x^2y^2 + 4y)dy = 0$$

$$\frac{\partial M}{\partial y} = 6xy^2 - 6x^3y^2 - 8xy \quad ; \quad \frac{\partial N}{\partial x} = 6xy^2$$

ليست تامة

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-6y^2x^3 - 8xy}{3x^2y^2 + 4y} = \frac{-2x(3x^2y^2 + 4y)}{3x^2y^2 + 4y}$$

$$= -2x \Rightarrow M = \int -2x dx = -x^2$$

$$\Rightarrow (2xy^3e^{-x^2} - 2x^3y^3e^{-x^2} + 2xe^{-x^2})dx + (3x^2y^2e^{-x^2} + 4ye^{-x^2})dy$$

$$(2xe^{-x^2}dx) + (4ye^{-x^2}dy - 4xye^{-x^2}dx) + (3x^2y^2e^{-x^2})dy$$

$$+ (2xy^3e^{-x^2} - 2x^3y^3e^{-x^2})dx = 0$$

$$-e^{-x^2} + 2y^2e^{-x^2} + x^2y^3e^{-x^2} = C$$

$$② (y + 1 + x^2)dx + (x^2 \cos y - x)dy = 0$$

$$\frac{\partial M}{\partial y} = 1 \quad , \quad \frac{\partial N}{\partial x} = (2 \cos(y))x - 1$$

المعادلة غير تامة

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - 2(\cos y)x + 1 = 2 - 2(\cos y)x$$

$$= -2(x \cos y - 1)$$

$$\frac{-2(x \cos y - 1)}{x(x \cos y - 1)} = -\frac{2}{x} \Rightarrow M = \int -\frac{2}{x} dx = -2 \ln x$$

$$= -2 \ln \frac{1}{x^2} \Rightarrow M = \frac{1}{x^2}$$

$$\frac{1}{x^2} (y+1+x^2) dx + \frac{1}{x^2} (x^2 \cos y - x) dy = 0$$

$$\left(\frac{y}{x^2} + \frac{1}{x^2} + 1\right) dx + \left(\cos y - \frac{1}{x}\right) dy = 0$$

$$\left(\frac{1}{x^2} + 1\right) dM + \cos y dy + \left(\frac{y}{x^2} dx - \frac{1}{x} dy\right) = 0$$

$$-\frac{1}{x} + x + \sin y - \frac{y}{x} = C$$

$$\Rightarrow (x^3 + y^3) dx - xy^2 dy = 0 \quad (3)$$

$$\frac{\partial M}{\partial y} = 3y^2 \quad \text{و} \quad \frac{\partial N}{\partial x} = -y^2$$

ليست تامة تفصل على تكامل من التكامل

$$M = \frac{1}{x^4 + y^3 x - xy^3} = \frac{1}{x^4}$$

$$\frac{1}{x} dx + \frac{y^3}{x^4} dx - \frac{y^2}{x^3} dy = 0$$

بالنسبة لـ $\frac{y^2}{x^3}$ بالنسبة لـ $\frac{y^3}{3x^3}$

$$\ln|x| + \frac{y^3}{3x^3} = C$$

$$(4) \quad y(1+xy)dx + x(1-xy)dy = 0$$

$$\frac{\partial M}{\partial y} = 1+2xy \quad \& \quad \frac{\partial N}{\partial x} = 1-2xy$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

المعادلة ليست متكافئة

$$y f(x,y)dx + x g(x,y)dy = 0$$

$$M = \frac{1}{xM - yN} = \frac{1}{xy(1+xy) - xy(1-xy)} \Rightarrow M = \frac{1}{2x^2y^2}$$

$$\frac{y(1+xy)}{2x^2y^2}dx + \frac{x(1-xy)}{2x^2y^2}dy = 0$$

$$\frac{1}{2x^2y}dx + \frac{1}{2x}dx + \frac{1}{2xy^2}dy - \frac{1}{2y}dy = 0$$

$$\left(\frac{1}{2x}dx\right) + \left(-\frac{1}{2y}dy\right) + \left(\frac{1}{2xy}dx + \frac{1}{2xy^2}dy\right) = 0$$

$$\frac{1}{2} \ln|x| - \frac{1}{2} \ln|y| - \frac{1}{2xy} = C$$

$$(y^4 + 2y)dx + (xy^3 - 2y^4 - 4x)dy = 0 \quad (5)$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2 \quad \& \quad \frac{\partial N}{\partial x} = y^3 - 4$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} = 3y^2 + 6$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{-3y^3 + 6}{-(y^4 + 2)} = -\frac{3}{y} \Rightarrow M = e^{\int -\frac{3}{y} dy} = \frac{1}{y^3}$$

$$(y + \frac{2}{y^2})dx + (x - 2y - \frac{4x}{y^3})dy = 0$$

$$(y dx + x dy) + (\frac{2}{y^2} dx - \frac{4x}{y^3} dy) - 2y dy = 0$$

$$y x + \frac{2}{y^2} x - y^2 = C$$

$$y dx + (1 - 3x^3 y^3) dy = 0 \quad (6)$$

$$\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = 1 - 9x^3 y^3 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - (1 - 9x^3 y^3) = 9x^3 y^3$$

~~$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$~~

نفرض $w = x^2 y^2$

$$\frac{\partial w}{\partial x} = 2xy^2; \frac{\partial w}{\partial y} = 2yx^2$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{9x^3 y^3}{2xy^2(1 - 3x^3 y^3) - 2yx^2(y)} dy \Rightarrow \frac{\partial M}{M} = -\frac{3}{2} \left(\frac{1}{x^2 y^2} \right) dw$$

$$\frac{\partial M}{M} = -\frac{3}{2} \left(\frac{1}{w} \right) dw \Rightarrow \ln |M| = -\frac{3}{2} \ln |w|$$

$$\ln |M| = \ln(w)^{-\frac{3}{2}} \Rightarrow \frac{y}{x^2 y^3} dx + \left(\frac{x}{x^3 y^3} - \frac{3x^3 y^3}{x^3 y^3} \right) dy$$

$$= \frac{1}{x^3 y^2} dx + \left(\frac{1}{x^2 y^3} - 3 \right) dy$$

$$= \left(\frac{1}{x^3 y^2} dx + \frac{1}{x^2 y^3} dy \right) - 3 dy$$

$$\int \frac{x^{-3}}{y^2} dx = \frac{x^{-2}}{-2y^2} = \frac{1}{-2x^2 y^2}$$

$$\frac{1}{2x^2 y^2} - 3y = C$$

$$(7). (x^2 - y^2 + 2x) dx + (x^2 - y^2 - 2y) dy = 0$$

$$\frac{\partial M}{\partial y} = -2y$$

$$\frac{\partial N}{\partial x} = 2x \Rightarrow \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -2y - 2x = -2(x+y)$$

$$\Rightarrow W = x+y \text{ انفر}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

$$\frac{-2(x+y)}{N - M} = \frac{-2(x+y)}{-(x^2 - y^2 + 2x - x^2 + y^2 + 2y)} = \frac{-2(x+y)}{-2(x+y)} = 1$$

$$P \int dw = e^w = e^{x+y}$$

$$(x^2 e^{x+y} - y^2 e^{x+y} + 2x e^{x+y}) dx + (x^2 e^{x+y} - y^2 e^{x+y} - 2y e^{x+y}) dy$$

$$= x^2 e^{x+y} - y^2 e^{x+y} + 2x e^{x+y} + x^2 e^{x+y} - y^2 e^{x+y} - 2y e^{x+y}$$

$$[(x^2 e^{x+y} + 2x e^{x+y}) dx + (x^2 e^{x+y}) dy] + [(-y^2 e^{x+y}) dx + (-y^2 e^{x+y} - 2y e^{x+y}) dy] = (x^2 e^{x+y} - y^2 e^{x+y}) = C$$