كلية العلوم

القسم : **الرياضيات**

السنة : **الثالثة**

المادة : معادلات تفاضلية جزئية

المحاضرة : الثانية/عملي/

{{ A to Z مكتبة }}

حل :

① $-3u_x + u_t = 0$

$u(x,0) = e^{x^3}$

$$\frac{dx}{-3} = \frac{dt}{1} = \frac{du}{0}$$

$$\boxed{x + 3t = k_1}$$

$$\boxed{u(x,t) = k_2}$$

$F(x+3t, u) = 0$

$\Rightarrow u = g(x+3t)$

$u(x,0) = g(x) = e^{-(x+3t)^2}$

② $x u_x + y u_y = x e^{-u}$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{x e^{-u}}$$

2 و 1 $\Rightarrow$ $\boxed{\frac{x}{y} = k_1}$

3 و 1 $\Rightarrow$ $dx = e^u du$

$$\boxed{x - e^u = k_2}$$

$$F\left(\frac{x}{y},\ x-e^{u}\right)=0$$

$$x-e^{u}=g\left(\frac{x}{y}\right)$$

$$e^{u}=x-g\left(\frac{x}{y}\right)$$

$$\Rightarrow u=\ln\left(x-g\left(\frac{x}{y}\right)\right)$$

③ $u_x+u_y-u=y$

$$\frac{dx}{1}=\frac{dy}{1}=\frac{du}{y+u}$$

$2 \& 1 \Rightarrow \boxed{x-y=k_1}$

$2+3=2 \Rightarrow$ $$\frac{dy}{1}=\frac{du+dy}{1+u+y}$$

$$y=\ln|1+u+y|-\ln k_2$$

$$\frac{1+u+y}{k_2}=e^{y}$$

$$\Rightarrow \boxed{(1+u+y)e^{-y}=k_2}$$

$$F\left(x-y,\ (1+u+y)\,e^{-y}\right)=0$$

$$(1+u+y)\,e^{-y}=g(x-y)\Rightarrow u=e^{y}g(x-y)-1-y$$

4) $yu_x + xu_y = x^2 + y^2$

$u(x,0) = 1 + x^2$

$u(0,y) = 1 + y^2$

$$\frac{dx}{y} = \frac{dy}{x} = \frac{du}{x^2+y^2}$$

$$\frac{x^2}{2} - \frac{y^2}{2} = \frac{k_1}{2} \Rightarrow \boxed{x^2 - y^2 = k_1}$$

$-y(1) - x(2) + 3 \Rightarrow \dfrac{-y\,dx - x\,dy + du}{-y^2 - x^2 + x^2 + y^2}$

نعدم المقام ⇐

$$-y\,dx - x\,dy + du = 0$$

$$-[y\,dx + x\,dy] + du = 0$$

$$-d(xy) + du = 0$$

بالمكاملة: $\boxed{-xy + u = k_2}$

$$F(x^2 - y^2, -xy + u) = 0$$

$$-xy + u = f(x^2 - y^2)$$

$$\Rightarrow u = f(x^2 - y^2) + xy$$

الشرط الأول:

$$u(x,0) = f(x^2) + 0 = 1 + x^2$$

$f(x^2) = 1 + x^2 \Rightarrow f(x) = 1 + x \quad ; x \geq 0$

~~f(x) = 1+1+x~~

الشرط الثاني:

$f(-y^2) + 0 = u(0,y) = 1 + y^2 \Rightarrow f(y) = 1 - y \quad ; y \leq 0$

$\Rightarrow \quad u = 1 + |x^2 - y^2| + xy$

⑤ $u_{tt} = 9\, u_{xx}$

$u(0,x) = \cos x$

$u_t(0,x) = 0$

$u_{tt} - 9\, u_{xx} = 0$

$(D_t^2 - 9 D_x^2)\, u = 0$

$(D_t + 3D_x)(D_t - 3D_x)\, u = 0$

$u = f_1(x - 3t) + f_2(x + 3t)$

الشرط الأول :

$u(0,x) = f_1(x) + f_2(x) = \cos x \qquad (1)$

الشرط الثاني

$u_t = -3 f_1'(x - 3t) + 3 f_2'(x + 3t)$

$\Rightarrow u_t(0,x) = -3 f_1'(x) + 3 f_2'(x) = 0$

$-f_1'(x) + f_2'(x) = 0$

$C = -f_1(x) + f_2(x) \qquad (2)$

$\boxed{1+2} \Rightarrow \quad f_2(x) = \frac{1}{2}\cos x + \frac{1}{2} C$

$\boxed{-1+2} \Rightarrow \quad f_1(x) = \frac{1}{2}\cos x - \frac{1}{2} C$

$u = \frac{1}{2}\left[\cos(x - 3t) - C + \cos(x + 3t) + C\right]$

$\Rightarrow u = \frac{1}{2}\left[\cos(x - 3t) + \cos(x + 3t)\right]$

6 $u_{tt} = u_{xx}$

$u(x,0) = \frac{1}{1+x^2}$

$u_t(x,0) = 0$

$u_{tt} - u_{xx} = 0$

$(D_t^2 - D_x^2)u = 0$

$(D_t - D_x)(D_t + D_x)u = 0$

$u = f_1(x+t) + f_2(x-t)$

الشرط الأول:

$\frac{1}{1+x^2} = u(x,0) = f_1(x) + f_2(x)$ (1)

الشرط الثاني:

$u_t = f_1'(x+t) - f_2'(x-t)$

$\Rightarrow 0 = u_t(x,0) = f_1'(x) - f_2'(x)$

$f_1(x) - f_2(x) = c$ (2)

$\boxed{1+2} \Rightarrow f_1(x) = \frac{1}{2}\left[\frac{1}{1+x^2} + c\right]$

$f_2(x) = \frac{1}{2}\left[\frac{1}{1+x^2} - c\right]$

$u = \frac{1}{2}\left[\frac{1}{1+(x+t)^2} + c + \frac{1}{1+(x-t)^2} - c\right]$

$u = \frac{1}{2}\left[\frac{1}{1+(x+t)^2} + \frac{1}{1+(x-t)^2}\right]$

انتهت المحاضرة