كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : معادلات تفاضلية٢

المحاضرة : الثامنة/عملي/

{{ مكتبة A to Z }}

Facebook Group : مكتبة A to Z

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر(What's app-Telegram)على الرقم 0931497960

أوجد الحل العام للمعادلات الآتية:

(1) $(D^2 - DD' - 6D'^2)Z = e^{2x-3y}$

$$Z_h = f_1(y+3x) + f_2(y-2x)$$

$$Z_p = \frac{1}{D^2 - DD' - 6D'^2} e^{2x-3y} = \frac{-1}{44} e^{2x-3y}$$

$$Z = f_1(y+3x) + f_2(y-2x) - \frac{1}{44} e^{2x-3y}$$

(2) $(D^2 - 4DD' + 4D'^2)Z = \sin(2x-3y)$

$$(D-2D')^2 Z = \sin(2x-3y)$$

$$Z_h = f_1(y+2x) + x f_2(y+2x)$$

$$Z_p = \frac{1}{D^2 - 4DD' + 4D'^2} \sin(2x-3y)$$

$$Z_p = \frac{1}{-4-24-12} \sin(2x-3y) = \frac{-1}{40} \sin(2x-3y)$$

$$Z = f_1(y+2x) + x f_2(y+2x) - \frac{1}{40} \sin(2x-3y).$$

(3) $(D^2 - DD' - 6D'^2)Z = e^{3x+y}$

$$(D-3D')(D+2D')\,Z = e^{3x+y}$$

$$Z_h = f_1(y+3x) + f_2(y-2x)$$

$$Z_p = \frac{1}{(D-3D')(D+2D')}\,e^{3x+y}$$

$$Z_p = \frac{1}{(D-3D')}\cdot\frac{1}{D+2D'}\,e^{3x+y}$$

$$Z_p = \frac{1}{D-3D'}\cdot\frac{1}{D+2D'}\,e^{3x+y}$$

$$Z_p = \frac{1}{5}\cdot\frac{1}{D-3D'}\,e^{3x+y}$$

$y = a-3x$

$$Z_p = \frac{1}{5}\int e^{3x+a-3x}\,dx = \frac{1}{5}\int e^{a}\,dx$$

$$Z_p = \frac{x}{5}\,e^{y+3x}$$

$$Z = f_1(y+3x) + f_2(y-2x) + \frac{x}{5}\,e^{y+3x}$$

(4) $(D^2-2DD')Z = x^3y$

$$D(D-2D')Z = x^3y$$

$$Z_h = f_1(y) + f_2(y+2x)$$

$$Z_p = \frac{1}{D(D-2D')}\,x^3y$$

$$Z_p = \frac{1}{D}\cdot\frac{1}{(D-2D')}\,x^3y$$

$\because\ y = a-2x$

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$$Z_p = \frac{1}{D}\int x^3(a-2x)\,dx$$

$$Z_p = \frac{1}{D}\int (ax^3 - 2x^4)\,dx$$

$$Z_p = \frac{1}{D}\left(\frac{ax^4}{4} - \frac{2}{5}x^5\right)$$

$$Z_p = \frac{1}{D}\left(\frac{(y+2x)x^4}{4} - \frac{2}{5}x^5\right) \quad \because y=b$$

$$Z_p = \int \left(\frac{bx^4}{4} + \frac{2x^5}{4} - \frac{2}{5}x^5\right)dx$$

$$Z_p = \frac{bx^5}{20} + \frac{x^6}{12} - \frac{2x^6}{30}$$

$$Z_p = \frac{yx^5}{20} + \frac{x^6}{60}$$

$$\boxed{Z = f_1(y) + f_2(y+2x) + \frac{yx^5}{20} + \frac{x^6}{60}}$$

(5) $(D - D' + 2)Z = e^{2x+y}$

$$Z_h = e^{-2x} f_1(y+x)$$

$$Z_p = \frac{1}{D - D' + 2} e^{2x+y} = \frac{1}{3} e^{2x+y}$$

$$Z = e^{-2x} f_1(y+x) + \frac{1}{3} e^{2x+y}$$

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⑥ $(x^2D^2 + xD - yD - y^2D'^2)Z = y$

نفرض أن: $X = \ln x \quad , \quad Y = \ln y$

$xD = B \quad , \quad x^2D^2 = B^2 - B$

$yD' = B' \quad , \quad y^2D'^2 = B'^2 - B'$

نعوض في المعادلة نجد أن:

$(B^2 - B + B - B' - B'^2 + B')Z = e^y$

$(B^2 - B'^2)Z = e^y \Rightarrow (B - B')(B + B')Z = e^y$

$Z_h = f_1(Y + X) + f_2(Y - X)$

$Z_p = \frac{1}{B^2 - B'^2} e^y = e^y$

$Z = f_1(Y + X) + f_2(Y - X) - e^y$

$Z = f_1(\ln y + \ln x) + f_2(\ln y - \ln x) - y$

$Z = f_1(\ln xy) + f_2(\ln \frac{y}{x}) - y$

$Z = g_1(xy) + g_2(\frac{y}{x}) - y$

النهاية

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