الوظيفة الأولى

السؤال الأول: أنجز التكاملات التالية

$$I_1 = \int x(x+1)\,(x-2)dx, \qquad I_2 = \int \left(\frac{x+\sqrt{x}}{x^2}\right)dx, \qquad I_3 = \int \left(\frac{2}{3\sqrt{x}} + e^{3x} + (11)^x\right)dx$$

$$I_4 = \int \left(sin2x + \frac{1}{cos^2 3x} + \frac{3}{1+x^2}\right)dx, \qquad I_5 = \int \frac{dx}{chx - shx}, \qquad I_6 = \int \frac{-x-1}{\sqrt{1+x^2}}dx$$

السؤال الثاني: أنجز التكاملات التالية

$$I_1 = \int \frac{x+1}{\sqrt{x^2+2x+3}}dx, \qquad I_2 = \int \frac{arcsin(x)}{\sqrt{1-x^2}}dx, \qquad I_3 = \int \frac{dx}{(1+x^2)\big(arctan(x)\big)^3}$$

$$I_4 = \int \left(cotx + \frac{1}{\sqrt{x^2-1}}\right)dx, \qquad I_5 = \int \frac{dx}{sinx}$$

السؤال الثالث: أنجز التكاملات التالية

$$I_1 = \int \frac{dx}{1-cosx}, \qquad I_2 = \int x\sqrt{1-x}\,dx, \qquad I_3 = \int \sqrt{x\sqrt{x\sqrt{x}}}\;dx$$

$$I_4 = \int \frac{7}{1+x^2} e^{arctan\,x}\,dx, \quad I_5 = \int \frac{dx}{e^{2x}+1}, \qquad I_6 = \int \frac{dx}{e^{-x}+\pi},$$

$$I_7 = \int \frac{sh4x}{3+ch4x}dx$$

$$I_8 = \int \frac{dx}{1-chx}$$

حلّ الوظيفة الأولى

السؤال الأول: أنجز التكاملات التالية

$$I_1 = \int x(x+1)\,(x-2)dx, \qquad I_2 = \int \left(\frac{x+\sqrt{x}}{x^2}\right)dx, \qquad I_3 = \int \left(\frac{2}{3\sqrt{x}} + e^{3x} + (11)^x\right)dx$$

$$I_4 = \int \left(sin2x + \frac{1}{cos^2 3x} + \frac{3}{1+x^2}\right)dx, \qquad I_5 = \int \frac{dx}{chx - shx}, \qquad I_6 = \int \frac{-x-1}{\sqrt{1+x^2}}dx$$

الحل:

$$I_1 = \int x(x+1)\,(x-2)dx = \int (x^3 - x^2 - 2x)\,dx = \frac{x^4}{4} - \frac{x^3}{3} - x^2 + c$$

$$I_2 = \int \left(\frac{x+\sqrt{x}}{x^2}\right)dx = \int \left(\frac{1}{x} + x^{-\frac{3}{2}}\right)dx = ln|x| - \frac{2}{\sqrt{x}} + c$$

$$I_3 = \int \left(\frac{2}{3\sqrt{x}} + e^{3x} + (11)^x\right)dx = \int \left(\frac{2}{3}x^{-\frac{1}{2}} + e^{3x} + (11)^x\right)dx$$
$$= \frac{4}{3}\sqrt{x} + \frac{1}{3}e^{3x} + \frac{(11)^x}{ln(11)} + c$$

$$I_4 = \int \left(sin2x + \frac{1}{cos^2 3x} + \frac{3}{1+x^2}\right)dx = -\frac{1}{2}cos2x + \frac{1}{3}tan3x + 3arctan(x) + c$$

$$I_5 = \int \frac{dx}{chx - shx} = \int \frac{chx + shx}{ch^2 x - sh^2 x}dx = \int (chx + shx)dx = shx + chx + c$$

$$I_6 = \int \frac{-x-1}{\sqrt{1+x^2}}dx = \int \frac{-x}{\sqrt{1+x^2}}dx - \int \frac{1}{\sqrt{1+x^2}}dx$$
$$= -\frac{1}{2}\int (2x)(1+x^2)^{-\frac{1}{2}}dx - \int \frac{dx}{\sqrt{x^2+1}}$$
$$= -\frac{1}{2} \times \frac{(1+x^2)^{\frac{1}{2}}}{\frac{1}{2}} - ln\left(x + \sqrt{x^2+1}\right) + c$$
$$= -\sqrt{1+x^2} - ln\left(x + \sqrt{x^2+1}\right) + c$$

السؤال الثاني: أنجز التكاملات التالية

$$I_1 = \int \frac{x+1}{\sqrt{x^2+2x+3}}dx, \qquad I_2 = \int \frac{arcsin(x)}{\sqrt{1-x^2}}dx, \qquad I_3 = \int \frac{dx}{(1+x^2)\big(arctan(x)\big)^3}$$

$$I_4 = \int \left(cotx + \frac{1}{\sqrt{x^2-1}}\right)dx, \qquad I_5 = \int \frac{dx}{sinx}$$

الحلّ:

$$I_1 = \int \frac{x+1}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int (2x+2)(x^2+2x+3)^{-\frac{1}{2}}dx = \frac{1}{2}\times\frac{(x^2+2x+3)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$
$$= \sqrt{x^2+2x+3}+c$$

$$I_2 = \int \frac{arcsin(x)}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}arcsin(x)\,dx = \frac{\big(arcsin(x)\big)^2}{2}+c$$

$$I_3 = \int \frac{dx}{(1+x^2)\big(arctan\,(x)\big)^3} = \int \frac{(arctan\,x)^{-3}}{(1+x^2)}dx = \frac{\big(arctan(x)\big)^{-2}}{-2}+c$$
$$= -\frac{1}{2\big(arctan(x)\big)^2}+c$$

$$I_4 = \int \left(cotx + \frac{1}{\sqrt{x^2-1}}\right)dx = \int \left(\frac{cosx}{sinx} + \frac{1}{\sqrt{x^2-1}}\right)dx = ln|sinx| + ln\left|x+\sqrt{x^2-1}\right|+c$$

$$I_5 = \int \frac{dx}{sinx} = \int \frac{dx}{2sin\left(\frac{x}{2}\right)cos\left(\frac{x}{2}\right)} = \int \left(\frac{sin^2\left(\frac{x}{2}\right)+cos^2\left(\frac{x}{2}\right)}{2sin\left(\frac{x}{2}\right)cos\left(\frac{x}{2}\right)}\right)dx$$
$$= \int \left(\frac{sin\left(\frac{x}{2}\right)}{2cos\left(\frac{x}{2}\right)} + \frac{cos\left(\frac{x}{2}\right)}{2sin\left(\frac{x}{2}\right)}\right)dx = -\int \frac{-\frac{1}{2}sin\left(\frac{x}{2}\right)}{cos\left(\frac{x}{2}\right)}dx + \int \frac{\frac{1}{2}cos\left(\frac{x}{2}\right)}{sin\left(\frac{x}{2}\right)}dx$$
$$= -ln\left|cos\left(\frac{x}{2}\right)\right| + ln\left|sin\left(\frac{x}{2}\right)\right| + c = ln\left|\frac{sin\left(\frac{x}{2}\right)}{cos\left(\frac{x}{2}\right)}\right| + c = ln\left|tan\left(\frac{x}{2}\right)\right| + c$$

السؤال الثالث: أنجز التكاملات التالية

$$I_1 = \int \frac{dx}{1-cosx}, \qquad I_2 = \int x\sqrt{1-x}\,dx, \qquad I_3 = \int \sqrt{x\sqrt{x\sqrt{x}}}\,dx$$

$$I_4 = \int \frac{7}{1+x^2}e^{arctan\,x}\,dx, \quad I_5 = \int \frac{dx}{e^{2x}+1}, \qquad I_6 = \int \frac{dx}{e^{-x}+\pi},$$

$$I_7 = \int \frac{sh4x}{3+ch4x}dx$$

$$I_8 = \int \frac{dx}{1-chx}$$

الحلّ:

$$I_1 = \int \frac{dx}{1-cosx} = \int \frac{(1+cosx)dx}{1-cos^2x} = \int \frac{(1+cosx)}{sin^2x}dx = \int \left(\frac{1}{sin^2x} + \frac{cosx}{sin^2x}\right)dx$$
$$= -cotx - \frac{1}{sinx} + c$$

$$I_2 = \int x\sqrt{1-x}\,dx = -\int -x\sqrt{1-x}\,dx = -\int (1-x-1)\sqrt{1-x}\,dx$$
$$= -\int (1-x)\sqrt{1-x}\,dx + \int \sqrt{1-x}\,dx = -\int (1-x)^{\frac{3}{2}}dx + \int (1-x)^{\frac{1}{2}}dx$$
$$= \frac{(1-x)^{\frac{5}{2}}}{\frac{5}{2}} - \frac{(1-x)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{5}\sqrt{(1-x)^5} - \frac{2}{3}\sqrt{(1-x)^3} + c$$

$$I_3 = \int \sqrt{x\sqrt{x\sqrt{x}}}\ dx = \int \sqrt{xx^{\frac{1}{2}}x^{\frac{1}{4}}}\ dx = \int \sqrt{x^{\frac{7}{4}}}\ dx = \int x^{\frac{7}{8}}\ dx = \frac{x^{\frac{7}{8}+1}}{\frac{7}{8}+1} + c$$
$$= \frac{8}{15}x\sqrt{x\sqrt{x\sqrt{x}}} + c$$

$$I_4 = \int \frac{7}{1+x^2}e^{arctan\,x}\,dx = 7\int \frac{1}{1+x^2}e^{arctan\,x}\,dx = 7e^{arctan\,x} + c$$

$$I_5 = \int \frac{dx}{e^{2x}+1} = \int \frac{e^{-2x}}{e^{-2x}+1}dx = -\frac{1}{2}\int \frac{-2e^{-2x}}{e^{-2x}+1}dx = -\frac{1}{2}ln|e^{-2x}+1| + c$$
$$= -\frac{1}{2}ln(e^{-2x}+1) + c$$

$$I_6 = \int \frac{dx}{e^{-x}+\pi} = \int \frac{e^x}{\pi e^x+1}dx = \frac{1}{\pi}\int \frac{\pi e^x}{\pi e^x+1}dx = \frac{1}{\pi}ln|\pi e^x+1| + c$$
$$= \frac{1}{\pi}ln(\pi e^x+1) + c$$

$$I_7 = \int \frac{sh4x}{3+ch4x}dx = \frac{1}{4}\int \frac{4sh4x}{3+ch4x}dx = \frac{1}{4}ln|3+ch4x| + c = \frac{1}{4}ln(3+ch4x) + c$$

$$I_8 = \int \frac{dx}{1-chx} = \int \frac{1+chx}{1-ch^2x}dx = -\int \frac{1+chx}{sh^2x}dx = \int \left(-\frac{1}{sh^2x} - chx(shx)^{-2}\right)dx$$
$$= cthx - \frac{(shx)^{-2+1}}{-2+1} + c = cthx + \frac{1}{shx} + c$$
