كلية العلوم

القسم : **الرياضيات**

السنة : **الثانية**

المادة : لغة تخصصية٢

المحاضرة : مقرر

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Allied Schools (Jaranwala Campus)

What are numbers in Math?

A number is a mathematical tool which is used in counting individual quantities, calculating and quantifying.

In general, decimal number system is used which consists of 10 digits, from 0 to 9.

A Binary Number is made up of only 0s and 1s.

- Example of decimal number is 7.
- Example of Binary number is 1001.

Number Systems:

Any number can be formed with the help of 10 digits.

i.e. 0,1,2,3,4,5,6,7,8,9

These numbers are called numerals and theses numerals are known as 'Arabic numeral'.

Base of a Number System:

The number of digits involved in a number system is called the base of that number system.

- If a number system involves only two digits 0,1, then base is 2.
- A number system in which 10 digits 0,1,2,3,4,5,6,7,8 and 9 are used; is a system with base 10.
- If a number system involves 8 digits called octal number system.

Number System with Base 2, 8 and 10:

1. Number system with base 2:

A number system formed by only two digits 0,1 is called binary system and its base is 2. This system is not used in everyday life apparently. But is very important system because it is used in all types of computers. Because computer store information in the form of 0 and 1.

X $(1111)_2 = (15)_{10}$

ii. Convert $(1470)_8$ into decimal

$1 \times 8^3 + 4 \times 8^2 + 7 \times 8^1 + 0 \times 8^0$

$= 1 \times 512 + 4 \times 64 + 7 \times 8 + 0 \times 1$

$= 512 + 256 + 56 + 0$

$= 824$

$(1470)_8 = (824)_{10}$

Various Operations Performed on Numbers

Addition: It is the process of finding out single number or fraction equal to two or more quantities taken together.

For Example: $1 + 2 = 3$

Subtraction: It is the process of finding out the quantity left when a smaller quantity (number/ fraction) is reduced from a larger one.

For Example: $4 - 2 = 2$

Multiplication: It signifies repeated addition. If a number has to be repeatedly added, then that number is multiplicand. The number of multiplicands considered for addition is multiplier. The sum of the repetition is the product.

For Example: $4 \times 2 = 8$ In addition form $4 + 4 = 8$.

Division: It is a reversal of multiplication. In this we find how often a given number called divisor is contained in another given number called dividend. The number expressing this is called the quotient and the excess of the dividend over the product of the divisor and quotient is called remainder.

For Example: $\frac{6}{2} = 3$

Types of Numbers:

~~an integer is natural if it is greater or equal to zero.~~
يكون العدد الصحيح طبيعياً إذا كان أكبر أو يساوي الصفر

i. Natural Numbers:

Counting numbers (also called natural numbers): The set of numbers beginning 1, 2, 3, 4, . . . and going on infinitely. إلى ما لا نهاية Finit محدود / متناه

- Zero is not in this group.
- Set of natural number is denoted by N يرمز لها
- This group has no negative numbers.
- There are no numbers with decimals in this group.

Natural numbers are the set of positive integers, that is, integers from 1 to ∞ excluding fractional in decimal part. اعداد صحيحة

Set of natural number is: N = {1, 2, 3, 4, ...}.

The sum of two natural numbers is always a natural number. مجموع

For Example, 5 + 3 = 8 which is also natural.

The product of two natural numbers is also natural number. ناتج ضرب

For Example, 2 × 3 = 6 which is also natural number.

But when a natural number is subtracted from another natural number, the result is not always a natural number.

For Example, 3 – 5 = –2 and 3 – 3 = 0. Thus arose the concept of whole numbers and negative numbers.

ii. Whole Numbers: الأعداد الصحيحة

This group has all of the Natural Numbers in it plus the number 0.

They do not have any decimal or fractional part.

Whole number is denoted by W. يرمز لها

Set of whole number is: W = {0, 1, 2, 3, ...}.

Closure Property of Whole Numbers:

- Addition of whole numbers always gives a whole number. Thus whole numbers are closed under addition.
  - For Example, 3+8 = 11
- Subtraction of whole numbers does not always give a whole number. Thus whole numbers are not closed under subtraction.

- For Example, 5-7 = -2, which is a negative integer and not a whole number.
- For Example, 9-6 = 3 which is whole number.

- **Multiplication of whole numbers** always gives a whole number. Thus whole numbers are closed under multiplication.
  - For Example, 5x8 = 40
- **Dividing a whole number by another** does not always give a whole number. Thus whole numbers are not closed under division.
  - For Example, 4÷7 = a fraction, thus it will not be a whole number.

iii. **Integers:**

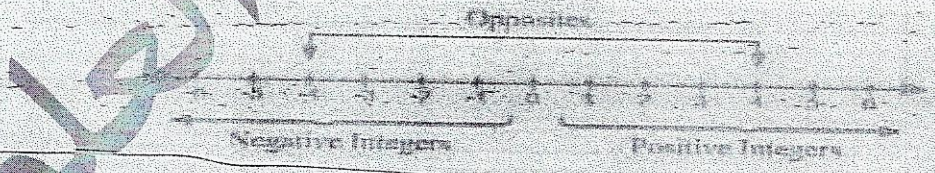
The set of counting numbers, zero, and negative counting numbers.

Set of Integer is denoted by Z

Set of integers is: $Z = \{0 \pm 1, \pm 2, \pm 3, 4, \ldots\}$

Integers are of two types:

- **Negative Integers**
  Negative integers are the set of negative numbers before 0. They do not have any fractional or decimal part. E.g. -1, -2, -3 and so on.
- **Positive Integers / Whole numbers**
  Positive integers / whole numbers are the set of natural numbers including zero. They do not have any fractional or decimal part.
- To distinguish between the positive and negative sides of 0, opposite signs are used, i.e. positive (+) and negative (-).



Additive Inverse:

The opposite of an integer is called its negative or additive inverse.

3 + (-3) = 0. So 3 is the additive inverse of -3 and vice versa.

Each integer on the number line, except 0, consists of its mirror image of the opposite sign. E.g. Mirror image of 6 will be -6, that of -1 will be 1 etc.

Zero is lesser than every positive integer and greater than every negative integer.

- Multiply their absolute values and prefix minus sign to the product.
- $2 \times (-3) = -6$

- **Multiplication of 2 negative integers:**
  - Multiply the absolute values and prefix plus sign to the product.
  - $(-2) \times (-3) = +6$ or 6

Division:

- ***Division of integers with like signs:***
  - Divide their absolute values and prefix plus sign.
  - $(-3) \div (-2) = +\frac{3}{2}$ or $\frac{3}{2}$
  - $(-18) \div (-6) = +3$ or 3
- ***Division of integers with like signs:***
  - Divide their absolute values and prefix minus sign.
  - $(3) \div (-2) = -\frac{3}{2}$

What is Zero?

Zero is the only whole number which is not a natural number. It is represented by 0.
"Zero is the only difference between natural and whole numbers."

Properties of ZERO (0)

- Zero added to any number gives the number itself. Hence, 0 is the additive identity for all numbers.
- Multiplication with zero: 0 multiplied with any number always gives 0.
- Zero divided by any number (except 0) gives 0.
- Any number divided by 0 is not defined.
- Zero to the power 0 (0^0) is not defined.

Natural Numbers can further be sub-divided into different types:

1. Even and odd Numbers
2. Prime Numbers
3. Relatively Prime/ co-prime numbers
4. Composite Numbers

Even Numbers:

- An even number is any number that can be divided by 2.
- For example, 12 can be divided by 2, so 12 is even.
- We saw in divisibility rules that a number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.
- Therefore, any number whose last digit is 0, 2, 4, 6, or 8 is an even number.
- Other examples of even numbers are 58, 44884, 998632, 98, 48, and 10000000.
- Formal definition of an even number:
- A number n is even if there exist a number k, such that n = 2k where k is an integer.
- This is formal way of saying that if n is divided by 2, we always get a quotient k with no remainder.
- Having no remainder means that n can in fact be divided by 2.

Odd Numbers:

- An odd number is any number that cannot be divided by 2.
- For example, 25 cannot be divided by 2, so 25 is odd.
- We saw in divisibility rules that a number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.
- Therefore, any number whose last digit is not 0, 2, 4, 6, or 8 is an odd number.
- Other examples of odd numbers are 53, 881, 238637, 99, 45, and 100000023.
- Formal definition of an odd number:
- A number n is odd if there exist a number k, such that n = 2k + 1 where k is an integer.
- This is formal way of saying that if n is divided by 2, we always get a quotient k with a remainder of 1.
- Having a remainder of 1 means that n cannot in fact be divided by 2.

Basic Operations with Even and Odd Numbers:

Addition:

even + even = even	For example: 2 + 4 = 6, Which is also even.
even + odd = odd	For example: 2 + 1 = 3, which is odd.

odd + odd = even For example: 5 + 3 = 8

Subtraction:

even – even = even For example: 8 – 4 = 4

even – odd = odd For example: 6 – 3 = 3

odd – odd = even For example: 9 – 5 = 4

Multiplication:

even × even = even For example: 2 × 4 = 8

even × odd = even For example: 4 × 3 = 12

odd × odd = odd For example: 5 ×3 = 15

Prime Numbers:

العدد الأول: يقبل القسمة على نفسه وعلى ① فقط ، بالتساوي

- A Prime Number can be divided evenly only by 1 or itself.
- And it must be a whole number greater than 1.
- For example, if we list the factors of 28, we have 1, 2, 4, 7, 14, and 28. That's six factors. If we list the factors of 29, we only have 1 and 29. That's two factors. So we say that 29 is a prime number, but 28 isn't.
- Note that the definition of a prime number doesn't allow 1 to be a prime number: 1 only has one factor, namely 1. Prime numbers have *exactly* two factors, not "at most two" or anything like that. When a number has more than two factors it is called a composite number.
- *Some Important points:*
- The only even prime number is 2. All other even numbers can be divided by 2.
- If the sum of a number's digits is a multiple of 3, that number can be divided by 3.
- No prime number greater than 5 ends in a 5. Any number greater than 5 that ends in a 5 can be divided by 5.
- Zero and 1 are not considered prime numbers.
- Except for 0 and 1, a number is either a prime number or a composite number. A composite number is defined as any number, greater than 1, that is not prime.

Co-Prime Numbers:

مجموعة الأعداد الأولية

عامل مشترك

- A set of numbers which do not have any other common factor other than 1 are called co-prime or relatively prime numbers.
- This means those numbers whose HCF is 1.

- For example, 8 and 9 have no other common factor other than 1 so they are co-prime numbers.
- ***Examples:***

Example 1: 21 and 22

- 21 and 22:
- The factors of 21 are 1, 3, 7 and 21.
- The factors of 22 are 1, 2, 11 and 22.
- Here 21 and 22 have only one common factor that is 1. Hence their HCF is 1 and are co-prime.

Examples 2: 21 and 27

- 21 and 27:
- The factors of 21 are 1, 3, 7 and 21.
- The factors of 27 are 1, 2, 3, 9 and 27.
- Here 21 and 27 have two common factors they are 1 and 3. HCF is 3 and they are not co-prime.

Rational Numbers:

Any number which can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$, is known as rational numbers.

For Example, $\frac{2}{3}, -\frac{3}{4}, -\frac{4}{7}, 0, 4, -2$

Hence positive integers, negative integers, zero and common fractions belong to the system of rational number.

The term rational is derived from the word 'ratio' because the rational numbers are figures which can be written in the ratio form.

Every whole number, including negative numbers and zero, is a rational number. This is because every whole number 'n' can be written in the form n/1.

- Examples of Rational Numbers
- The number 8 is a rational number because it can be written as the fraction 8/1.
- Likewise, 3/4 is a rational number because it can be written as a fraction.
- 1.5 is rational, because it can be written as the ratio 3/2.
- 0.333... (3 repeating) is also rational, because it can be written as the ratio 1/3.

a) A = {x : x is a factor of 60}

کیا ← ماضی ← لفظ

V_1	V_2	V_3
go	went	gone
break	broke	broken
buy	bought	bought
give	gave	given
eat	ate	eaten
write	wrote	written
read	read	read
drink	drank	drunk
teach	taught	taught
Come	Came	Come
become	became	become
drive	drove	driven
meet	met	met

V_1	V_2	V_3
Know	Knew	known
Take	took	taken
Speak	Spoke	Spoken
See	Saw	Seen
think	thought	thought
Sleep	Slept	slept
Swim	Swam	Swum
Steal	Stole	stolen
cut	Cut	Cut

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