



كلية العلوم

القسم : الفيزياء

السنة : الاولى

المادة : تحليل رياضي ٢

المحاضرة : الثانية / نظري /

{{ مكتبة A to Z }}

مكتبة A to Z : Facebook Group

كلية العلوم ، كلية الصيدلة ، الهندسة التقنية

يمكنكم طلب المحاضرات برسالة نصية (SMS) أو عبر (What's app-Telegram) على الرقم 0931497960

الدكتور :
المحاضرة:



القسم:
السنة:
المادة:

التاريخ:
المادة:

التاريخ: / /

A to Z Library for university services

$$I = \int \frac{dx}{x^4(4x^2)}$$

تحريث:

$$= \int \frac{1}{x^2(x^2+4)} dx$$

$$= \frac{1}{4} \int \frac{4}{x^2(x^2+4)} dx = \frac{1}{4} \int \frac{4+x^2-4x^2}{x^2(x^2+4)} dx$$

$$= \frac{1}{4} \int \left[\frac{4+x^2}{x^2(x^2+4)} - \frac{4x^2}{x^2(x^2+4)} \right] dx$$

$$I = \frac{1}{4} \int \left[\frac{1}{x^2} - \frac{1}{x^2+4} \right] dx$$

$$= \frac{1}{4} \int \left[x^{-2} - \frac{1}{x^2+4} \right] dx$$

$$= \frac{1}{4} \cdot \left[\frac{x^{-1}}{-1} - \frac{1}{2} \cdot \arctg \frac{x}{2} \right] + C$$

$$= -\frac{1}{4x} - \frac{1}{8} \arctg \frac{x}{2} + C$$

$$(2) \int \frac{\sqrt{x^2-3} - 3\sqrt{x^2+3}}{\sqrt{x^2-9}} dx$$

$$= \int \frac{\sqrt{x^2-3} - 3\sqrt{x^2+3}}{\sqrt{x^2-3}\sqrt{x^2-3}} dx$$

$$= \int \left[\frac{\sqrt{x^2-3}}{\sqrt{x^2+3}\sqrt{x^2-3}} - \frac{3\sqrt{x^2+3}}{\sqrt{x^2+3}\sqrt{x^2-3}} \right] dx$$

$$J = \int \left[\frac{1}{\sqrt{x^2+3}} - \frac{3}{\sqrt{x^2-3}} \right] dx$$

$$= \ln |x + \sqrt{x^2+3}| - 3 \ln |x + \sqrt{x^2-3}| + C$$

$$J = \ln |x + \sqrt{x^2+3}| - \ln |x + \sqrt{x^2-3}|$$

$$= \ln \left| \frac{x + \sqrt{x^2+3}}{x + \sqrt{x^2-3}} \right| + C$$

$$\textcircled{3} \quad K = \int \cos^2 \frac{x}{2} dx$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} = \frac{1}{2} + \frac{1}{2} \cos 2\theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2} \cos 2\theta$$

$$\Rightarrow K = \int \left[\frac{1}{2} + \frac{1}{2} \cos \left(\frac{x}{2} \right) \right] dx$$

$$K = \frac{1}{2}x + \frac{1}{2} \sin x + C$$

$$\begin{aligned}
 \textcircled{4} \quad I &= \int 3^x \cdot 5^{2x} dx \\
 &= \int 3^x (5^2)^x dx \\
 &= \int 3^x \cdot (25)^x dx = \int (3 \times 25)^x dx \\
 &= \int (75)^x dx = \frac{75^x}{\ln 75} + C
 \end{aligned}$$

$$\begin{aligned}
 \mu &= \int \frac{(x+1)(x^2-3)}{3x^2} dx \\
 &= \int \left[\frac{x^3 + 3x + x^2 - 3}{3x^2} \right] dx \\
 &= \int \left[\frac{1}{3}x - \frac{1}{x} + \frac{1}{3} - \frac{1}{x^2} \right] dx \\
 &= \left[\frac{1}{3} \frac{x^2}{2} + \frac{1}{3}x - \ln x + \frac{1}{x} + C \right] \\
 &= \frac{1}{6}x^2 - \ln x + \frac{1}{3}x + \frac{1}{x} + C
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad N &= \int \frac{(x+\sqrt{x})(1+\sqrt{x})}{3\sqrt{x}} dx \\
 &= \int \frac{2x + x\sqrt{x} + \sqrt{x}}{3\sqrt{x}} dx \\
 &= \int \frac{2x}{3\sqrt{x}} + \frac{x\sqrt{x}}{3\sqrt{x}} + \frac{\sqrt{x}}{3\sqrt{x}} dx
 \end{aligned}$$



$$= \int \left[2 \frac{x}{x^{\frac{1}{3}}} + \frac{x \cdot x^{\frac{1}{2}}}{x^{\frac{1}{3}}} + \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} + \frac{x}{x^{\frac{1}{3}}} \right]$$

$$= \int \left[x^{1-\frac{1}{3}} + x^{\frac{3}{2}-\frac{1}{3}} + x^{\frac{1}{2}-\frac{1}{3}} + x^{-\frac{1}{3}} \right]$$

$$= \int \left[2x^{\frac{2}{3}} + x^{\frac{7}{6}} + x^{\frac{1}{6}} + x^{-\frac{1}{3}} \right] dx$$

$$= 2 \cdot \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1}$$

$$= 2 \cdot \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{13}{6}}}{\frac{13}{6}} + \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C$$

$$= \frac{6}{5} \sqrt[3]{x^5} + \frac{6}{13} \sqrt{x^{13}} + \frac{6}{7} \sqrt{x^7}$$

طرق التكامل على الجدولة

① طريقة التحويل (طريقة تغيير متحول) تستخدم أحياناً

التكاملات من الشكل:

$$F = \int f(u(x)) \cdot dx$$

$$t = u(x)$$

تحويل

$$dx \cdot dt = u'(x) \cdot dx$$

$$\left(\frac{dx}{du} = dx \right)$$

نكتب I بدلالة t

$$I = \int \sqrt{x^3 - 3x^2} \cdot (x^2 - 2x) dx \quad \text{also}$$

$$t = x^3 - 3x^2 \quad \text{نقطة}$$

$$dt = (3x^2 - 6x) dx = 3(x^2 - 2x) dx$$

$$dx = \frac{dt}{3(x^2 - 2x)}$$

$$I = \int \sqrt{t} \cdot (\cancel{x^2 - 2x}) \cdot \frac{dt}{3(\cancel{x^2 - 2x})}$$

$$= \frac{1}{3} \int \sqrt{t} \cdot dt$$

$$= \frac{1}{3} \int t^{\frac{1}{2}} dt = \frac{1}{3} \cdot \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$I = \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3} \times \frac{2}{3} \cdot t^{\frac{3}{2}} + C$$

$$I = \frac{2}{9} \sqrt{t^3} + C$$

$$I = \frac{2}{9} \sqrt{(x^3 - 3x^2)^3} + C$$

$$J = \int \frac{1}{4x^2 + 4x + 2} dx = \int \frac{1}{4[x^2 + x] + 2} dx$$

$$= \int \frac{1}{4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 2} dx$$

$$= \int \frac{1}{4\left(x+\frac{1}{2}\right)^2 - 1 + 2} dx$$

$$= \int \frac{1}{(2)^2 \left(x+\frac{1}{2}\right)^2 + 1} dx$$

$$= \int \frac{1}{[2\left(x+\frac{1}{2}\right)]^2 + 1} dx = \int \frac{1}{(2x+1)^2 + 1} dx$$

$$t = 2x + 1 \quad \text{عوضه في المعادلة}$$

$$dt = 2 dx$$

$$dx = \frac{dt}{2}$$

$$\text{أي } t - 1 = 2x$$

$$\frac{1}{2}t - \frac{1}{2} = x$$

$$\frac{1}{2}dt = dx$$

$$J = \int \frac{1}{t^2 + 1} \times \frac{1}{2} dt$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 1} = \frac{1}{2} \times \frac{1}{1} \arctan t + C$$

$$= \frac{1}{2} \arctan(2x+1) + C$$



مكتبة
A to Z