كلية العلوم

القسم : **الفيزياء**

السنة : **الاولى**

المادة : تحليل رياضي٢

المحاضرة : **الثانية/نظري/**

{{ مكتبة A to Z }}

Facebook Group : مكتبة A to Z

تمرين:

$$I = \int \frac{dx}{x^4(4x^2)}$$

$$= \int \frac{1}{x^2(x^2+4)}\,dx$$

$$= \frac{1}{4}\int \frac{4}{x^2(x^2+4)}\,dx = \frac{1}{4}\int \frac{4+x^2-x^2}{x^2(x^2+4)}\,dx$$

$$= \frac{1}{4}\int \left[\frac{4+x^2}{x^2(x^2+4)} - \frac{x^2}{x^2(x^2+4)}\right]dx$$

$$I = \frac{1}{4}\int \left[\frac{1}{x^2} - \frac{1}{x^2-4}\right]dx$$

$$= \frac{1}{4}\int \left[x^{-2} - \frac{1}{x^2-4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^{-1}}{-1} - \frac{1}{2}\arctg\frac{x}{2}\right] + C$$

$$= -\frac{1}{4x} - \frac{1}{8}\arctg\frac{x}{2} + C$$

(2) $$J = \int \frac{\sqrt{x^2-3} - 3\sqrt{x^2+3}}{\sqrt{x^4-9}}\,dx$$

$$= \int \frac{\sqrt{x^2-3} - 3\sqrt{x^2+3}}{\sqrt{x^2-3}\sqrt{x^2-3}}\,dx$$

$$= \int \left[\frac{\sqrt{x^2-3}}{\sqrt{x^2+3}\sqrt{x^2-3}} - 3\frac{\sqrt{x^2+3}}{\sqrt{x^2+3}\sqrt{x^2-3}} \right] dx$$

$$J = \int \left[\frac{1}{\sqrt{x^2+3}} - \frac{3}{\sqrt{x^2-3}} \right] dx$$

$$= \ln\left|x + \sqrt{x^2+3}\right| - 3\ln\left|x + \sqrt{x^2-3}\right| + C$$

$$J = \ln\left|x + \sqrt{x^2+3}\right| - \ln\left|x + \sqrt{x^2-3}\right|$$

$$= \ln\left|\frac{x+\sqrt{x^2+3}}{x+\sqrt{x^2-3}}\right| = \ln\left|\frac{x+\sqrt{x^2+3}}{x+\sqrt{x^2-3}}\right| + C$$

(3) $$K = \int \cos^2\frac{x}{2}\, dx$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} = \frac{1}{2} + \frac{1}{2}\cos\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$$\Rightarrow K = \int \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{x}{x}\right) \right] dx$$

$$K = \frac{1}{2}x + \frac{1}{2}\sin x + C$$

(4) $I = \int 3^x \cdot 5^{2x} dx$

$= \int 3^x \cdot (5^2)^x dx$

$= \int 3^x \cdot (25)^x dx = \int (3 \times 25)^x dx$

$= \int (75)^x dx = \frac{75^x}{\ln 75} + C$

$M = \int \frac{(x+1)(x^2-3)}{3x^2} dx$

$= \int \left[\frac{x^3 + 3x + x^2 - 3}{3x^2} \right] dx$

$= \int \left[\frac{1}{3}x - \frac{1}{x} + \frac{1}{3} - \frac{1}{x^2} \right] dx$

$= \left[\frac{1}{3}\frac{x^2}{2} + \frac{1}{3}x - \ln x + \frac{1}{x} + C \right]$

$= \frac{1}{6}x^2 - \ln x + \frac{1}{3}x + \frac{1}{x} + C$

(5) $N = \int \frac{(x + \sqrt{x})(1 + \sqrt{x})}{\sqrt[3]{x}} dx$

$= \int \frac{2x + x\sqrt{x} + \sqrt{x}}{\sqrt[3]{x}} dx$

$= \int 2\frac{x}{\sqrt[3]{x}} + \frac{x\sqrt{x}}{\sqrt[3]{x}} + \frac{\sqrt{x}}{\sqrt[3]{x}}$

$$= \int \left[2\frac{x}{x^{\frac{1}{3}}} + \frac{x \cdot x^{\frac{1}{2}}}{x^{\frac{1}{3}}} + \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} + \frac{x}{x^{\frac{1}{3}}} \right]$$

$$= \int \left[x^{1-\frac{1}{3}} + x^{\frac{3}{2}-\frac{1}{3}} + x^{\frac{1}{2}-\frac{1}{3}} + x^{1-\frac{1}{3}} \right]$$

$$= \int \left[2x^{\frac{2}{3}} + x^{\frac{7}{6}} + x^{\frac{1}{6}} + c \right] dx$$

$$= 2\frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1}$$

$$= 2\frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{13}{6}}}{\frac{13}{6}} + \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + c$$

$$= \frac{6}{5}\sqrt[3]{x^5} + \frac{6}{13}\sqrt{x^{13}} + \frac{6}{7}\sqrt[6]{x^7}$$

طرق التكامل عند الجدة:

① طريقة التعويض (طريقة تغيير متحول) سنتعرف على

التكامل من الشكل:

$$F = \int f(\varphi(x))\, dx$$

نفرض $t = \varphi(x)$

عندئذ $dt = \varphi'(x)\, dx$

$$\frac{dt}{\varphi'(x)} = dx$$

نكتب I بدلالة t

مثال $I = \int \sqrt{x^3 - 3x^2} \cdot (x^2 - 2x)\, dx$

نفرض $t = x^3 - 3x^2$

$dt = (3x^2 - 6x)\, dx = 3(x^2 - 2x)\, dx$

$dx = \frac{dt}{3(x^2 - 2x)}$

$I = \int \sqrt{t} \cdot (x^2 - 2x) \cdot \frac{dt}{3(x^2 - 2x)}$

$= \frac{1}{3} \int \sqrt{t}\, dt$

$= \frac{1}{3} \int t^{\frac{1}{2}}\, dt = \frac{1}{3} \cdot \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$I = \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3} \times \frac{2}{3} \cdot t^{\frac{3}{2}} + C$

$I = \frac{2}{9} \sqrt{t^3} + C$

$I = \frac{2}{9} \sqrt{(x^3 - 3x^2)^3} + C$

$I = \int \frac{1}{4x^2 + 4x + 2}\, dx = \int \frac{1}{4[x^2 + x] + 2}\, dx$

$= \int \frac{1}{4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 2}\, dx$

$$= \int \frac{1}{4(x+\frac{1}{2})^2 - 1 + 2} dx$$

$$= \int \frac{1}{(2)^2(x+\frac{1}{2})^2 + 1} dx$$

$$= \int \frac{1}{[2(x+\frac{1}{2})]^2 + 1} dx = \int \frac{1}{(2x+1)^2 + 1} dx$$

نفرض أن $t = 2x+1$

$dt = 2 dx$

$dx = \frac{dt}{2}$ أي $t-1 = 2x$

$\frac{1}{2}t - \frac{1}{2} = x$

$\frac{1}{2} dt = dx$

$$J = \int \frac{1}{t^2+1} \times \frac{1}{2} dt$$

$$= \frac{1}{2} \int \frac{dt}{t^2+1} = \frac{1}{2} \times \frac{1}{1} \arctg t + c$$

$$= \frac{1}{2} \arctg(2x+1) + c$$